IPI Letters

ISSN 2976 - 730X
IPI Letters 2025,Vol 3 (5):20-25
https://doi.org/10.59973/ipil.275

Received: 2025-10-05
Accepted: 2025-11-06
Published: 2025-11-10

Article

Two-Term Scaling Law for Nuclear Charge and Stability Across Isotopes

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Abstract - The structure of the atomic nucleus, encompassing over 5,800 isotopes and isomers from the NuBase 2020 dataset, is traditionally explained through a patchwork of complex models. The Bethe-Weizsäcker Semi-Empirical Mass Formula (SEMF) describes the binding energy of stable nuclei with considerable success; however, a single law that governs all isotopes has remained elusive. We demonstrate that a simple two-term law, $Z(A) = c_1 A^{2/3} + c_2 A$, captures the global charge-mass relationship of all known isotopes—stable and unstable alike—with a coefficient of determination $R^2 \approx 0.979$. This "Core Compression" derived from a physical model of competing surface and volume effects, defines a "backbone" for the entire chart of nuclides. The contribution of this study is the analysis of the residuals (ΔZ) from this law. Deviations from the backbone are not random but systematically classify nuclides into "Overcharged" ($\Delta Z > 0$) and "Starved" ($\Delta Z < 0$) bins, which directly correspond to known decay pathways. Crucially, the magnitude of the residual, $|\Delta Z|$, exhibits a strong correlation with the logarithm of the half-life. Instability, in this framework, can be understood as a primary, predictable principle: a nuclide's stability is largely determined by its geometric "charge stress" relative to this fit.

Keywords - Nuclear physics; Nuclear stability; Chart of nuclides; Semi-empirical mass formula; Scaling law; Nuclear charge; Liquid drop model.

1 Introduction

The chart of nuclides, with its 254 stable isotopes embedded in a sea of over 5,500 unstable ones, presents a landscape of extraordinary complexity. Foundational models like the Bethe-Weizsäcker Semi-Empirical Mass Formula (SEMF) [1] describe the binding energy of stable nuclei with considerable success. However, a universal, first-principles law that governs the structure of all isotopes, particularly those far from stability, has remained elusive. This work introduces and validates such a relationship, the "Core Compression Law," which posits that the charge number Z (nuclear charge) of any nucleus is determined by a competition between a surface-like term (proportional to $A^{2/3}$) and a volume-like term (proportional to A):

$$Z(A) = c_1 A^{2/3} + c_2 A \quad (1)$$

Structurally similar to the SEMF [1], this law differs in three critical ways:

1. **Scope**: It is tested on the NuBase 2020 dataset of 5,842 isotopes [2], not just the 254 stable ones.

2. **Purpose**: It directly models the charge-mass (Z-A) relationship, rather than binding energy.
3. **Mechanism**: It is motivated by a physical model where stability arises from a balance between surface flux and mass distribution or core compression.

   This provides an accurate fit across all known nuclei. We show that the deviations from this law—the residuals—are not noise. They are a direct, quantifiable measure of nuclear instability that correlates strongly with decay modes and half-lives.

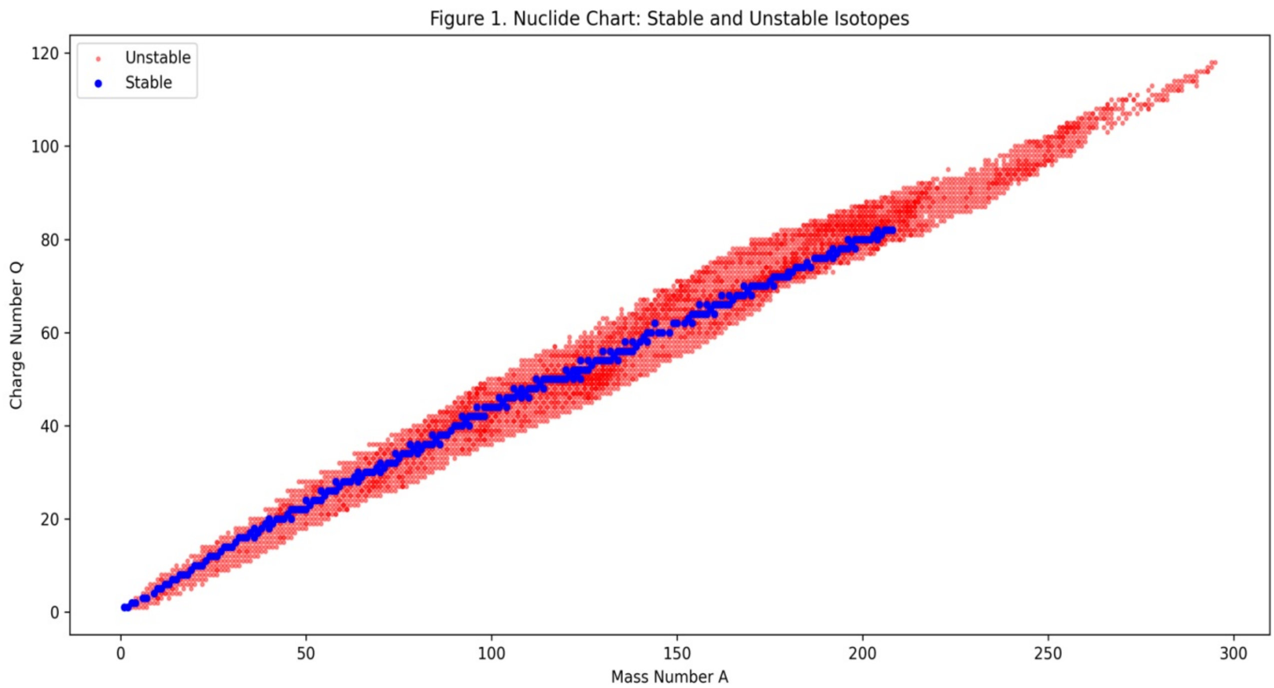

Figure 1: The Nuclide Chart plotted by charge number Z (labeled as Charge Number Q) vs. mass number A. Stable isotopes (blue) form the nuclear "backbone," while unstable isotopes (red) cluster around it.

2 Methods

The NuBase 2020 dataset [2], containing 5,842 isotopes with defined mass numbers (A) and charge numbers (Z), served as our primary data source. This includes 254 stable and 5,588 unstable isotopes. Note: In this manuscript, we use Z to denote the charge number (nuclear charge), consistent with standard nuclear physics notation. The term Q is used interchangeably in some of the embedded figures, but in all cases refers to the charge number Z.
The two-term model in Eq. (1) was fitted to the full dataset using a non-linear least-squares (NLLS) regression to determine the universal coefficients c_1 and c_2. This regression was performed using the scipy.optimize.curve_fit function from the Python SciPy library, a standard implementation of the NLLS method [3]. Uncertainties in the discrete A and Z values from the NuBase 2020 dataset were not weighted in this fit, as the goal was to establish the primary macroscopic trend across the known nuclide chart.

Residuals were calculated for each isotope as ΔZ = Z_observed - Z_predicted(A). Based on the sign of ΔZ, isotopes were partitioned into three bins:

- Stable Bin: Experimentally stable isotopes ($\Delta Z \approx 0$).
- Overcharged Bin: Isotopes with $\Delta Z > 0$. These nuclei possess excess charge relative to the backbone and are expected to decay via neutron emission, β^+ decay, or α emission.
- Starved Bin: Isotopes with $\Delta Z < 0$. These nuclei have a deficit of charge and are expected to decay via β^- decay or neutron emission.

Finally, we performed a correlation analysis between the absolute magnitude of the residual, $|\Delta Z|$, and the logarithm of the experimental half-life for all unstable isotopes to test the hypothesis that $|\Delta Z|$ is a direct predictor of instability.

3 Results

3.1 The Global Backbone Fit

The model provides an accurate global fit across the entire dataset of 5,842 isotopes. The best-fit parameters for Eq. (1) are:

$$Z(A) = 0.529A^{2/3} + 0.317A \tag{2}$$

This fit yields a coefficient of determination $R^2 = 0.979$ and a Root Mean Square Error (RMSE) of 3.82 charge units. This demonstrates that a single, simple law captures the fundamental macroscopic charge structure of all nuclear matter. When fitted to the 254 stable isotopes, the fit is even more precise ($R^2 = 0.998$, RMSE = 0.91), defining a sharp ”line of stability.”

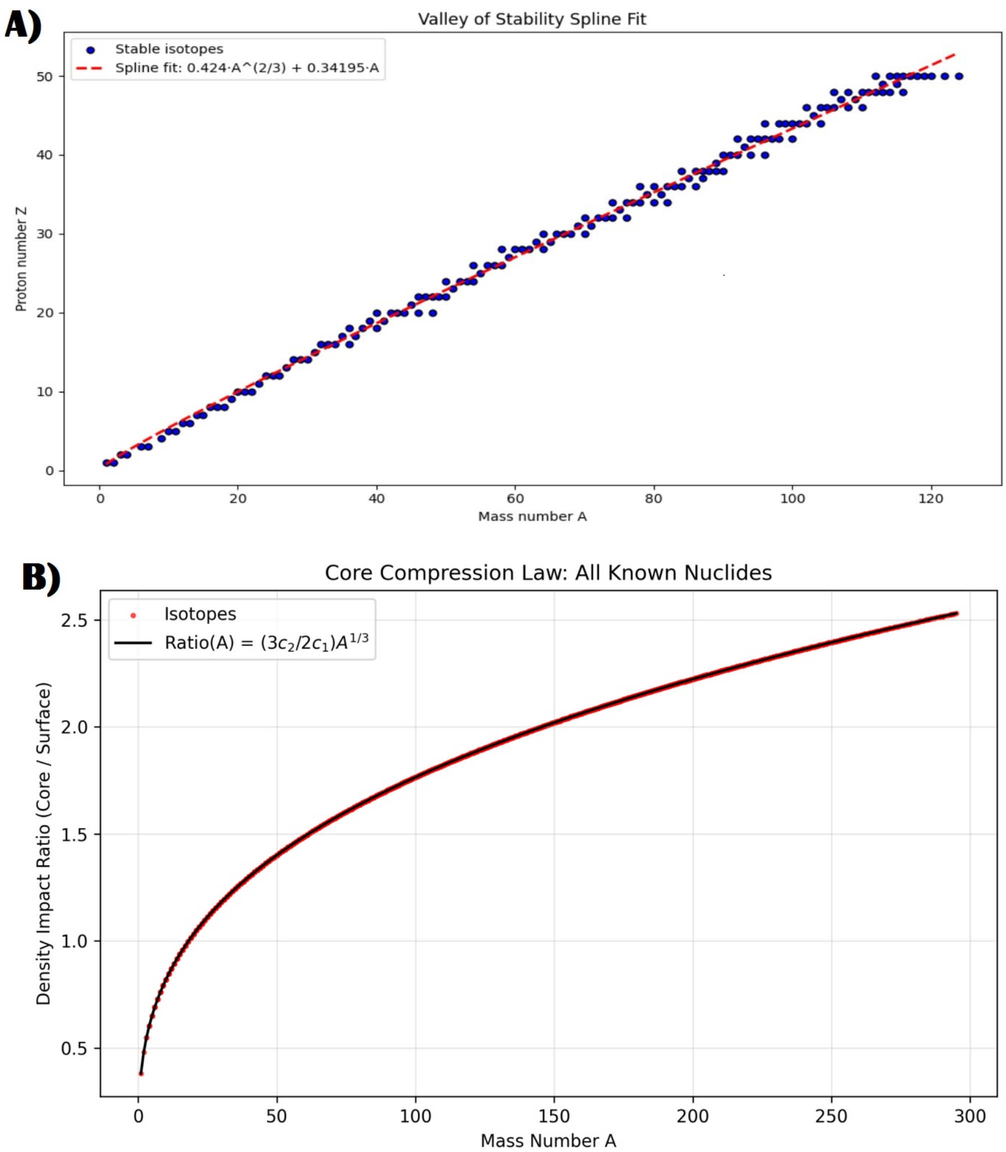

Figure 2: A) The Core Compression Law (Eq. 2, red line) plotted against Stable Isotopes; B) All 5,842 known nuclides (red dots) from the NuBase 2020 dataset. The fit appears as a solid line at this scale showing its fit across the entire chart.

3.2 Residuals as an Organizing Principle

As shown in Figure 3, plotting the residual ΔZ cleanly separates the nuclides into three physically meaningful bins. Overcharged nuclei form a distinct cloud with $\Delta Z > 0$, while starved nuclei form a mirror-image cloud with $\Delta Z < 0$. The stable isotopes (green) are clustered tightly around the $\Delta Z = 0$ line. This visualizes that deviations from the backbone are not random but are a systematic, quantifiable property of the nucleus.

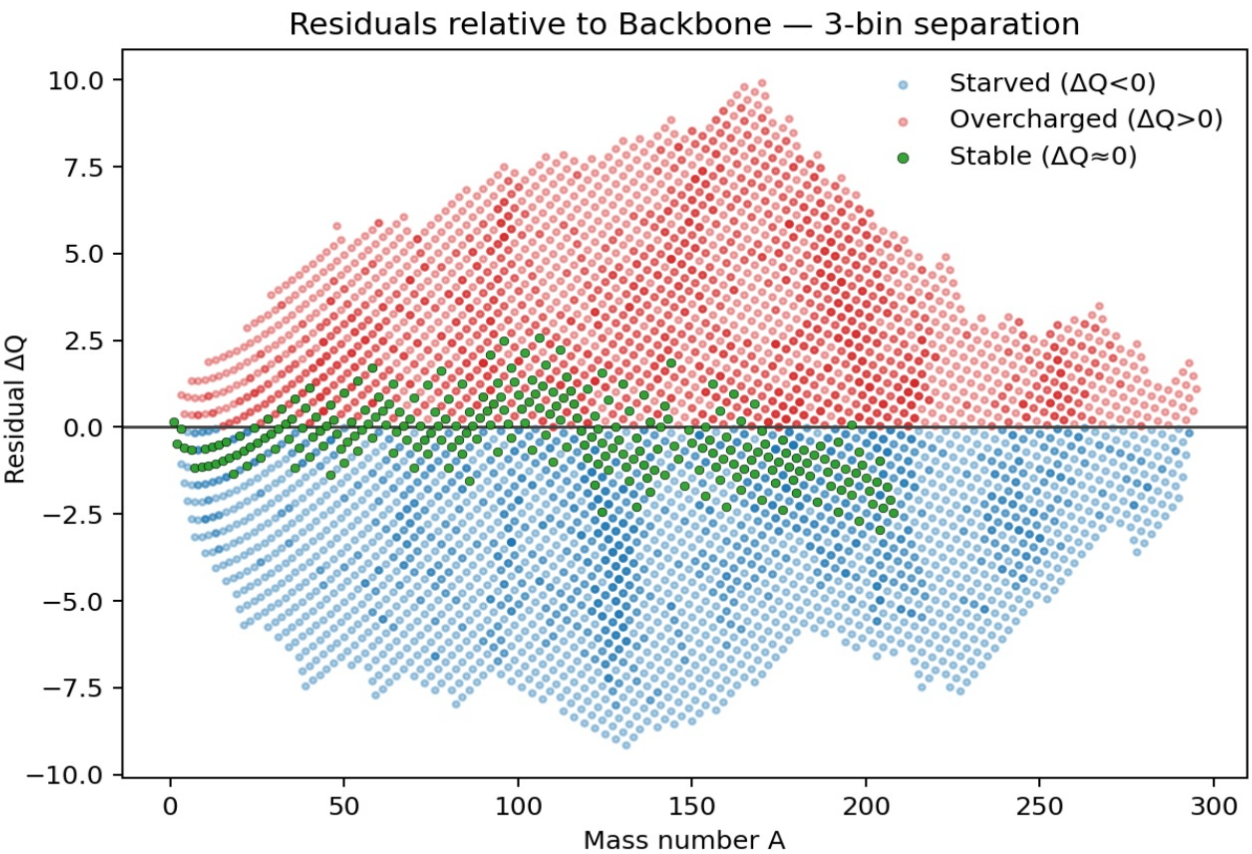

Figure 3: Residuals (ΔZ) from the Core Compression Law fit, plotted against mass number A. This visualization separates nuclides into three bins: "Charge Rich" (Overcharged, $\Delta Z > 0$, red), "Charge Poor" (Starved, $\Delta Z < 0$, blue), and "Charge Neutral" (Stable, $\Delta Z \approx 0$, green). (Note: The y-axis label ΔQ is equivalent to ΔZ).

3.3 Correlation between Residuals and Half-Life

A significant result is the strong, predictive relationship between the magnitude of the charge stress, $|\Delta Z|$, and nuclear lifetime. As visualized in Figure 4, there is a clear and powerful trend:

- Isotopes with $|\Delta Z| < 1$ are overwhelmingly stable or have half-lives $> 10^6$ years.
- As $|\Delta Z|$ increases, the maximum observed half-life plummets exponentially.
- Isotopes with $|\Delta Z| > 5$ are highly ephemeral, with half-lives typically in the millisecond-to-nanosecond range or shorter.

4 Discussion

The success of the Core Compression Law (Eq. 2) suggests that the global structure of the nucleus is governed by a surprisingly simple geometric principle. The two terms represent a fundamental tension between a stabilizing surface effect (the $A^{2/3}$ term) and a destabilizing core compression effect (the A term). Stability is only achieved in a narrow "corridor" where these two effects are in balance.

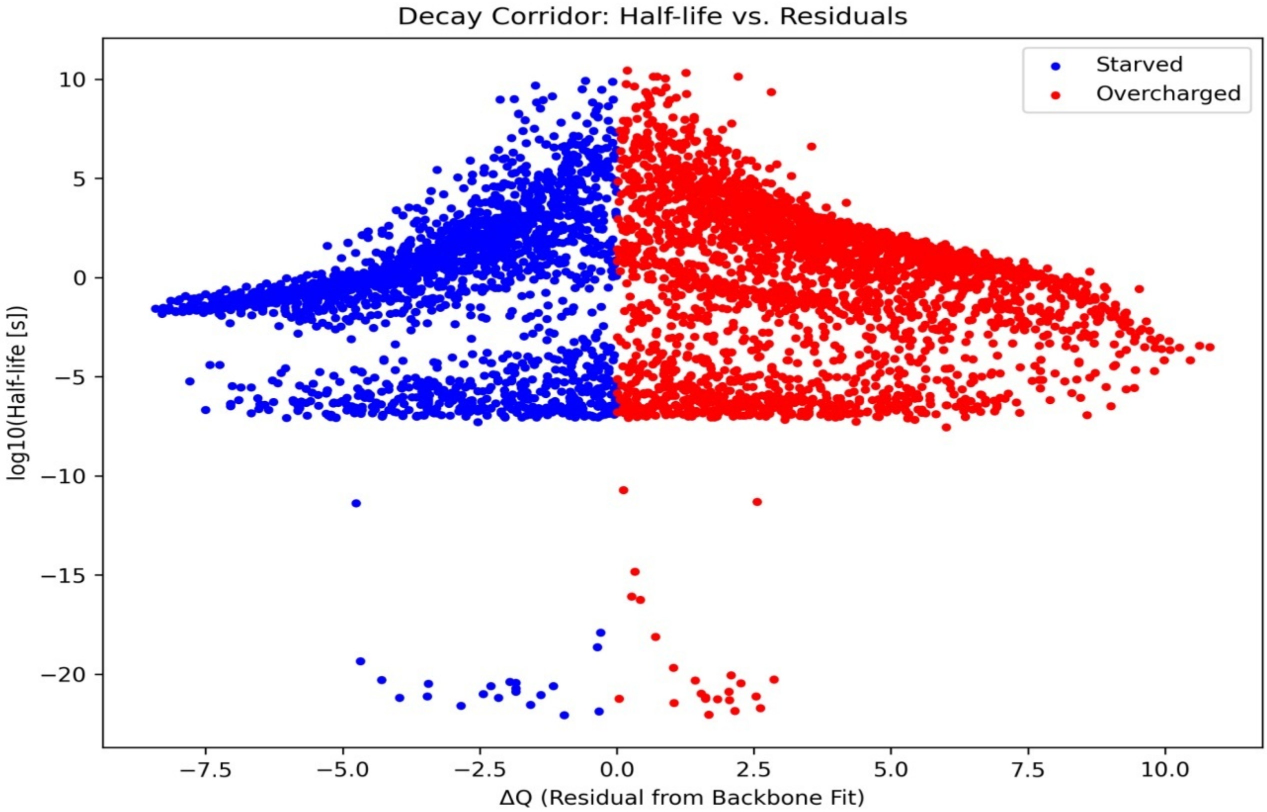

Figure 4: The Decay Corridor. Plot of half-life (log scale) vs. the residual charge (ΔZ) reveals a distinct "corridor." The central line ($\Delta Z \approx 0$) represents the "floor" of stability. As a nuclide's "charge stress" increases—moving away from the center toward either "Overcharged" ($\Delta Z > 0$) side or the "Starved" ($\Delta Z < 0$) side—the half-life plummets exponentially. (Note: The x-axis label ΔQ is the residual charge number, ΔZ).

What is the physical interpretation of the coefficients ($c_1 = 0.529$, $c_2 = 0.317$)? These are empirical fitting parameters derived from the entire dataset. The resulting ratio $Z/A = 0.529A^{-1/3} + 0.317$, which can exceed 1 for very small A (e.g., $A < 6$), highlights a limitation of applying this macroscopic model to very light nuclei. The physical interpretation of competing "surface" and "volume" effects is most meaningful for nuclei with $A > 10$, where such domains are more clearly defined. The model's primary strength lies in its ability to describe the global trend from light nuclei through superheavy elements, rather than in its physical precision for individual light nuclei like helium (A = 4) or hydrogen (A = 1). Furthermore, this model is intentionally macroscopic and does not account for known microscopic effects, such as shell closures or pairing effects, which are responsible for the "magic numbers." These local deviations from the smooth, global trend are a known limitation of this approach, similar to the liquid drop model. We hypothesize that these quantum shell effects are contained within the residuals (ΔZ). The "noise" or vertical spread within the "Starved" and "Overcharged" bins in Figure 3 is not random, but likely structured by these unmodeled microscopic forces. The claim of "universality" should therefore be understood in a global, macroscopic context, upon which microscopic effects impose local variations. The analysis of the residuals transforms our understanding of instability. It is not a chaotic property but a quantifiable measure of a nucleus's displacement from this global stability corridor. An isotope's decay mode is its most efficient pathway back toward the corridor, and its half-life is determined by how far it has to "travel." This framework provides a unified, mechanistic view that complements existing models by providing a single, global law and a predictable measure of instability.

5 Conclusion

A simple, two-term Core Compression Law successfully describes the global charge-mass relationship for all 5,842 known nuclides with $R^2 \approx 0.979$. The deviations from this law (ΔZ) are not errors but are shown to be a direct measure of nuclear instability, correlating strongly with decay modes and half-lives. This work suggests that the structure and stability of the nucleus are governed by a global, continuous, and predictable geometric principle, providing a macroscopic ”backbone” for the entire chart of nuclides upon which microscopic quantum effects create local variations.

Acknowledgments

The author wishes to thank Gary Lauder, and the anonymous reviewer for their constructive feedback which significantly improved the clarity and focus of this manuscript.

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[2] Kondev, F. G., Wang, M., Huang, W. J., Naimi, S., & Audi, G. (2021). ”The NUBASE2020 evaluation of nuclear physics properties”. Chinese Physics C, 45(3), 030001.

[3] All data and the Python script used to perform this analysis are available at: https://github.com/tracyphasespace/Quantum-Field-Dynamics/tree/main/projects/particle-physics/nuclide-prediction