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Invited Article

Extending the Standard Model through Generation Number Reinterpretation

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Abstract - It is often stated that there is little obvious relationship between the rest mass energies in the standard model except that mass increases with generation in each one of the particle types, i.e. within the series of quarks, electron leptons and possibly neutrinos considered individually. However, this is misleading. A clearer, more coherent, pattern emerges when (a) the notion of a generation number is treated as a quantum number divorced from separately considering each series, (b) when the particles are considered as arranged by the rest mass plus a small constant mass in the manner of a loop quantum field correction (with focus on the "package" that somehow holds the particles of similar mass), and (c) when the logarithms of the mass-energies are plotted in a particular way against the generation number to include the photon, Higgs particle, W and Z bosons, and a popular form of the conjectured graviton. The fact that the convenient use of log MeV/c^2 mass energy is not dimensionless is discussed. Three mainstreams I, II, III emerge from the clusters; the first two differ in slope and extrapolate to converge persuasively at a generation zero that is tempting to associate with massless spin 0 bosons, notably the photon, generating 11 clusters of mass energies with reasonably tight dispersion. It is suggested that the 4 generations may arise through a phase shift of $\pi/2$ related to 4 orthogonal superpositions of spins or bits |0> and |1> comprehensible in terms of axis rotations corresponding to progressive multiplication of the superpositions by imaginary number i. A projected mass energy for mainstream III, primarily the neutrino series, projected to generation zero, lies in the mass domain of axion-like particles (ALPs), still considered good dark matter candidates. The amplitude is more problematic, but, Chester's pedagogical bead-on-a-circular-wire model provides a plausible explanation in terms of wavenumbers and kinetic energy as mass energy. The discussion here is not so much to propose a specific theory, but rather to help stimulate some ideas. It remains that there seems to be more pattern in the rest masses of the fundamental particles than usually described, and that seems worthy of investigation.

Keywords - Standard model; Generation number; Rest masses; Fermions; Bosons.

1 Introduction - The Standard Model and Particle Rest Masses

The Standard Model of Particle Physics does not alone predict the values of the masses of fundamental particles. An underlying more quantitative, *ab initio*, model as a basis for the values of the masses is still lacking (e.g., Refs [1-3]). In the approaches taken to resolve this, theoretical physicists have proposed the possibility of more than three generations of stable fermions many times, particularly in the contexts beyond the Standard Model (BSM) theories. Experimental evidence disfavors a fourth generation of standard, chiral fermions. This may, however, be missing the point. A generation number 0 could be associated with fundamental particles that have the character of bosons, as discussed in this paper, such as the photon, and other widely studied hypothetical particles. In preon theories (1970s–1980s) discussed briefly here, quarks and leptons are composites of more fundamental particles. Some versions include bosonic preons, which might be seen as a kind of "zeroth layer" from which all fermions emerge. The problem is that this and other models do not map directly to a generation scheme, and only conceptually resemble the idea. A slightly different view of masses in respect to generation numbers could help resolve that.

Some degree of introduction from a less usual perspective may be helpful in the context of the present paper, which discusses that problem. The Dirac Equation $(i\gamma^{\mu}\partial\psi) = \mathrm{m}\psi$ is a famous relativistic equation that relates mass m to a wave function ψ of a spinor field in spacetime coordinates x, y, z, and t, and its corresponding derivatives $\partial\,\psi$, with a relationship described by certain gamma matrices γ^{μ}. However, it does not predict mass, which is seen as an input parameter. Dirac did not emphasize this equation nor any details of Quantum Field Theory in his book [4] because it focused on basic non-relativistic quantum mechanics, but he does give early mention in it of a linear operator σ (Chapter 2, equation 23) such that $\sigma\sigma = +1$, which allows *dualization* of the wave amplitude in terms of operator expressions ½(1+σ) and ½(1-σ). They would later be seen as spinor projectors, or a flavor of them with σ as γ^5 (one of the above-mentioned γ matrices) [5]. As taught by Penrose [5], mass can arise by reference to the above dualization into spinors, seeing them as giving rise to a frequency or "zigzag" between the spinor projections that is "essentially" related to the de Broglie frequency [5]. Penrose does not at that point mention the Higgs mechanism as involved, though he does subsequently discuss mass as likely to arise from engagement of the zigzag with the Higgs field. The later experimental validation of the existence of a Higgs boson [6] supported the Higgs mechanism now seen as the primary source of mass at least for W bosons, Z bosons, and some fermions, through chiral symmetry breaking of spinor projection and electroweak symmetry breaking. However, all the above is qualitative as far as predicting mass values is concerned: each has a different empirical coupling constant that still lacks a persuasive quantitative explanation.

It is widely assumed that there must be further guidance in the distributions of the rest mass energies of stable particles of the Standard Model in considering properties such as generation number, charge, and spin (and isospin involving the arrangement of quarks where protons and neutrons are concerned). However, it is also widely held that there is yet no quantitative pattern even in the distribution of particle masses plotted against the value of a chosen property. It is of course well known that particles such as quarks and massive leptons (the electron series) do increase across the three generations, and logarithms of rest mass energies for, e.g., the electron-muon-tau and up-charm-top quark series do appear to follow monotonic convex curves, but even there the increases across these generations are 200- and 17-fold and 13- and 130-fold respectively. There have been many persistent efforts over several decades to find a clearer pattern, such as that of Koide [7]. More recent informal efforts have indicated a roughly exponential increases in masses taken collectively *when plotted against ranked order*. For example, Walters [8] notes that when the scattering energies, in the customary experimental physicist's measure of mass in GeV/c^2 where c is the velocity of light, are plotted against ranked order n of mass values, there is an initial rapid rise converging to a linear slope that then tends off somewhat to a plateau or shallower slope, the *overall* natural logarithmic fit being to a line -14.38+1.243n with a significant standard error of 4.31. Note that his plot, like that in the present paper, included the neutrino series using only the upper limits of recent best estimates of the possible masses: so far, the absolute experimental masses in that series have not been identified. Of course, more is known about the mass ratios between neutrinos than the absolute values of their masses. The use of rank order of particle masses and the roughly logarithmic relationships might well mean that this is somehow related, at least in part, to a much more general distribution law such as Zipf's law [9]. This is likely to be more of statistical than physical significance. For example, it applies to many phenomena including size of cities ranked by size, or relative frequency of words used in a language ranked by usage.

2 Classes of Model

There are several models that represent different approaches to quantify rest-mass energy. The above zigzag and Higgs mechanism is the mainstay of the Standard Model, but as it is at present, it is a mechanism only, not a method of calculating mass values. Methods with more quantitative aspirations are Supersymmetry (mass relations among partners), GUTs (possible mass ratios), Preon Models (mass predicted from substructure), Scale-Invariant Models (mass via symmetry breaking), and closer to those discussed in the present paper, Extra Dimensions Models (mass from compactification), and String Theory which principle could generative quantitative estimates of absolute mass, but like the rest has failed to do so. Detailed discussion of these is beyond present scope (though see, e.g. Refs [1-3,5,10]) but, conceptually at least, efforts to approach computation of particle mass in the Standard Model fall into classes. The first class includes extensions of the "zigzag and Higgs" approach to tackling the particle mass problem sees rest mass energy primarily as an oscillation between two states creating "mass energy in frequency", E = hν. There the symbols have their usual meaning as energy, Planck's constant, and frequency, and more specifically in a de Broglie interpretation in which, in the case of interest in the present paper, frequency is primary, and mass energy is emergent from it [5]. The oscillation does not relate to a physical circle, but represents a quantum particle, particularly a massive fermion like an electron, described as a superposition of two components that are themselves massless, each moving at the speed of light in opposite directions; that oscillation is between left-handed and right-handed chiral (Weyl) spinor states [5]. This is consistent with the fact that massless fermions (e.g. neutrinos in the Standard Model before mass terms were added) have a fixed chirality and only move in one direction. That is, they zig or they zag, but not both. Although this has all provided great insight it has yet failed to account for the quantitative values of the particle masses on an *ab initio* basis.

There are other classes of model. For example, it is commonly held that for the protons and neutrons, any engagement of the Higgs mechanism is less direct, and that most mass comes from the energy of the strong interaction (gluon fields) between their composite quarks, and that binding energy and motion of quarks and gluons inside hadrons account for roughly 95% of visible mass in the universe [3]. Many models form a class in which the origins of rest mass are considered as more fundamentally residing not in some notion of zigzag frequency but rather in a locally confined kinetic energy, as the form E = mc^2 analogous to E = 1/2 mv^2 suggests. It is tempting to dismiss this as duality, a different perception of essentially the same thing, with rest mass energy quantization arising from wave number $2\pi\nu$/v, i.e. arising from some intrinsic mass moving at velocity v, perhaps a photon at velocity c. However, an essential difference of the second class of model is that here is specific confinement in a relatively small volume of space by some kind of "package" formed by a curled up spatial dimension. In contrast to the zigzag and Higgs that is essentially a kind of particle oscillation, it could mean a kind of circle in some sense. That is still a very general description. It could imply, for example, (a) a string of string theory, open or closed [10], (b) an entity analogous to a photon "packet" joined head to tail such that the wave packet is periodic in space, forming a mode sin(kx-ωt) where k is the wave number ω is the angular frequency, x is the position, and t is the time) where the spatial domain is a circle, or (c) a circular "curled up" dimension in which a wave packet could otherwise run freely and fully in the form of a discrete packet. Such models are essentially the efforts going beyond the Standard Model listed above, such Grand Unified Theories (GUTs), and often implying the use of flavor symmetry groups such as SU(3). Because the distinguishing feature of models of this class is that they reflect some kind of circular topology as a circular path, and so they might reasonably be collectively described as "orbital models". In many cases, they do clearly relate to the wave number $2\pi \nu$/v discussed above, with the wave function determining an energy spectrum in which the energies as seen as mass energies. The problem here is in accounting for the distribution of energy levels, as follows.

3 Background to the Present Study

The current paper arose from the application of data mining and AI applied to stable, unstable, and hypothetical particles and the properties such as charge, spin, and isospin, with the original intension of exploring introduction of other descriptors derived from theoretical physics. Usually, the important quantum numbers are considered to be the principal quantum number (n), the azimuthal (or angular momentum) quantum number (l), the magnetic quantum number (m_l), and the spin quantum number (m_s). The method will be described elsewhere, because the outcome is reasonably simple to describe as a proposal for forming a model. Initial studies reduced to considering the masses along with 3 descriptive parameters as independent variables seen as dimensions and elements of arrays in a purely mathematical sense: (i) the generation number 1-3 with the option to extend beyond 3 generations if required, (ii) 6 x the absolute value of electrical charge, and (iii) 2 x (the absolute value of) the spin. For example, the electron is represented by [1,6,1] having a mass of 0.511 MeV, and the bottom quark from [3,2,1], from which empirical descriptions the correlation with mass is to be computed by linear regression described below. Note that, for example, the 3 in [3,2,1] means that it is in a generation 3 from the quark series even though that places it close to the mass of the tau particle [3,2,1] which could be said to be primarily (but by no means solely) placed by the resulting analysis in the massive lepton (electron) series. One reason for the outcome being simple is that the method did not lead to any reasonable formulas for predicting the masses (including non-additive formulas where contributions are interdependent). Nonetheless, a clearer pattern emerged as shown in Fig. 1 by plotting the data in a particular way. This simply involves plotting the logarithm (here the natural logarithm) of the experimental rest mass energy in MeV/c^2 (Mega electron volts divided by the speed of light squared as used in scattering studies) plus a small constant (see below) against the generation number without thinking in terms of the masses on a separate family-by-family (series-by-series, fermion-by-fermion, or boson-by-boson) basis. In other words, generation number 1-3 is retained as a quantum state property on a par with charge, spin, etc. and is thus divorced from individual consideration of generations as applying to the group particles of the same type. In the Standard Model, the generation number does not appear in the Lagrangian as a quantum number and is essentially a label, but some extensions beyond the Standard Model that were mentioned in Section 2, i.e. as Grand Unified Theories (GUTs), use of flavor symmetry groups such as SU(3), and string theory and extra-dimensional models. Notably, it is consistent with the general class of package model considered in Section 2 where mass energy is dictated by its appearance as a wave packet or behavior in curled-up dimensions, characterized by coincidence of, say, wave number, irrespective of how that wave number arises from other properties.

The addition of a small constant to all experimental mass energies (but arguably with an implied off-figure exception discussed later below in Section 6) involves going beyond the Standard Model in which some particles as taken as massless. Adding a small constant to the rest mass is consistent with correction in loop quantum field theory, where particles can emit and reabsorb virtual particles, even if they are not directly observable (which could be relevant analogy to theory underling the present case). It is relevant that loop correction is an example of situations in which logarithmic plots of masses and energies can arise. The loop corrections regarding mass are described as the factor m_{trace} in "bare mass" m → physical mass as $m_{rest} + m_{trace}$ where each value of m_{rest} is the widely accepted experimental rest mass energy in convenient MeV from scattering studies. In Fig. 1, this is expressed in terms of plotting the convenient natural logarithm MeV/c^2 of mass energy $E = mc^2$ corrected as $m_{trace} + m_{rest}$ with $m_{trace} = 0.020$ MeV/c^2.

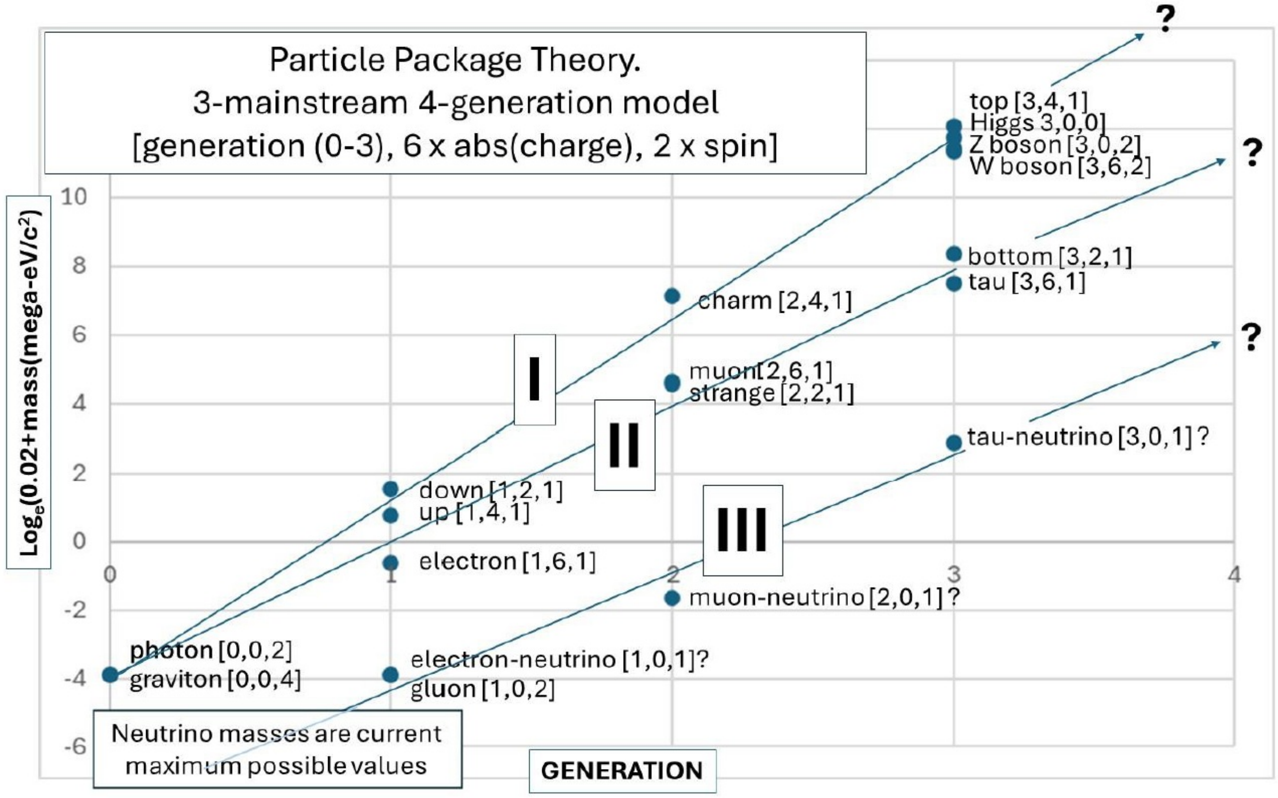

Figure 1: The Natural Logarithm of 0.020 plus mass in MeV/c^2 versus Generation Number.

This value is a result of best fit considering the extrapolations discussed below. This addition of m_{trace} was also motivated by the "orbital models" of Section 2 in mind. For example, a photon moving in a tight circle still has zero rest mass, but the system overall can behave as if it has an effective mass or energy due to confinement. This is in any case of interest here one of the origins or particle rest mass considered below. If a photon is confined to moving in a circle, say as in an optical cavity or along a waveguide loop, one is modifying the boundary conditions but not the fundamental properties of the photon. The system containing the photon (e.g. a cavity with circulating light) can have perceived nonzero invariant mass.

Clearly, there are certain features relating to the placing of certain particles in Fig. 1 that are to various extents conjectural, and they are there to focus discussion later below, introduced in Section 3, but arguably, for the most part, Fig. 1 is empirical, not theoretical, and not controversial. It is of course well understood that several different particles from very different families, such as top quark, Higgs boson, W^+, W^- and Z^0 boson, form well-defined clusters of similar mass, i.e. relatively little dispersion, on a logarithmic scale of mass, and similar clusters are also formed from particles that are not obviously related. The apparent exception of adding m_{trace} to the experimental mass is, as noted above, not novel from the perspective of loop quantum theory, and importantly a low value of $m_{trace} = 0.020$ Mev/c^2 does not significantly affect the masses of massive particles in the *established* three generations. Importantly, it is also determined empirically by the extrapolation of log mass energy trends as follows. For the established generation numbers of 1, 2, and 3, there are 9 clusters of masses with a sufficiently small dispersion to consider them as distinct clusters. For those clusters, the role of the charge and spin seems, at this stage, only to be that a cluster of masses cannot contain particles with same charge and spin. However, there is the additional observation that 9 initial and 11 final clusters lie, to reasonable approximation, on three log-linear mainstreams called I, II, and III. The term ”mainstream” is borrowed from astrophysics: a Hertzsprung-Russell (H-R) diagram is a graph that plots stars based on their magnitude (brightness) and temperature. Unlike the H-R case, Fig. 1 shows three mainstreams that *relate* respectively to the quark, massive lepton (electron) and neutrino series (recall, neutrino masses are upper limits, though relative values are considered reliable from neutrino oscillation experiments). However, by no means do they relate exactly in the case of mainstreams I and II. The term ”mainstream” seems appropriate because there are particles, accepted and predicted, which would lie well off these lines, and that is so in the extreme case of the assumed zero mass of the photon and recognized models for the graviton before considering interpretation of the clusters and extrapolation of mainstreams I and II. It is the extension and intersection of mainstreams I and II from generation numbers 1, 2, and 3 opens the possibility of the existence of a generation number 0 associated with a m_{trace} = 0.020 MeV / c^2 from the established masses even before 0.020 MeV / c^2 is added to the experimental masses. Interpreting that intersection as a generation number 0 is because the extrapolations intersect on the generation zero axis. The neutrino family generation numbers 1, 2, and 3 of mainstream III are extrapolated to the generation number 0 axis *after* adding this correction mass, but nothing is assumed as to a mass correction like m_{trace} when the regression line cuts that zero axis at about -8 units of log MeV/c^2, i.e. at a mass energy of *circa* 0.000335 MeV/c^2.

4 Considerations in the Exploration of Possible Models

As noted above, there are clearly certain features in Fig. 1 that are to various extents conjectural, and that largely relates to the placing of certain particles by name. Some justifications for these assignments are discussed later (Section 6), but certain aspects impact the choice of models. That the photon and favored form of the graviton lie at the intersection of mainstreams I and II and represent particles with generation number 0 is conjectural, but the intersection at 0 on a generation number axis conceptually begs to be populated by particles and otherwise such bosons are lacking any placing on the diagram. Moreover, the above addition of the m_{trace} = 0.020 is a model that requires that they sit along the -4 log mass energy line assuming that the linear log extrapolations are meaningful. As partners in the same cluster, the photon and preferred graviton model are conceptually similar in that they are both gauge bosons rooted in field theory with a scalar field of charge zero. There an initial assumption was that spin 0 required (though it is not always associated with) low mass, indeed no mass at all in the original Standard Model.

The other and arguably much more conjectural feature is the location of a gluon mass at generation number 1 alongside the neutrino, based on an original speculation that there a colorless gluon that does exist and corresponds to generation number 0. That speculation has been discarded, but the placing of the gluon alongside the maximum possible mass of the generation 1 neutrino is primarily based on the requirement of gauge symmetry that it is massless, albeit here except for m_{trace}= 0.020 Mev/c^2 being added. While the generation 1 neutrino is not required by gauge theory to be massless, and is not believed to be so, it is the electron neutrino that is closest to being massless by the relative values known from neutrino oscillation experiments. If a gluon had the mass of a muon or tau neutrino it would likely dramatically weaken or destroy its ability to bind quarks together.

Having once assigned and populated a generation number 0 with the photon and graviton as above, it is natural to consider also placing some particle or entity and the point of intersection of the relative masses of the neutrino family with the generation number 0 axis. As noted above, the neutrino family generation numbers 1, 2, and 3 in Fig. 1 are extrapolated *after* adding this correction mass, but nothing is assumed as to a mass correction similar in concept to m_{mtrace} when the regression line cuts that zero axis at about -8 units of log MeV/c^2, i.e. at a mass energy of circa 0.000335 MeV/c^2. This is an approximate value of mass energy widely assumed to be associated with a light sterile neutrino LSN, though this is very light compared to typical sterile neutrino models (which more often place these particles in the keV range). It could correspond to a hypothetical light boson or dark photon, if its interaction cross-section is extremely small. It is too light to be standard cold dark matter, but it could be considered "warm" or even "hot" dark matter. It would affect structure formation on small scales. The mass of any sterile neutrino is not really known except as theoretical parameter that depends on the model: proposed models give masses anywhere from a few eV up to several GeV or more [1-3]. VLNs are particles considered as having masses in the range 0.000001–0.00001 MeV/c^2, which would put them in the range -11 down to -14 on the generation number 0 axis. There is a conflict here because, in many models, sterile neutrinos are considered as hypothetical *heavier* neutrinos that do not interact via the weak force. Consequently, placing them as having a much lower mass energy than even the electron-neutrino while remaining in mainstream III seems to make little sense, at least if the extrapolation is to be taken as the basis for predicting the presence of the particle. The linear extrapolation would have to be broken, and the mainstream curve bent upwards at that point, whatever the absolute mass of the electron neutrino.

Any candidate particle extrapolated from mainstream III (neutrinos) and placed near the linear log intercept of -8 MeV/mc^2 would also be a candidate for a "very light boson" and very light bosonic dark matter, so in effect a warm dark matter candidate. In actuality, 0.000335 MeV/c^2 would be considered rather low for consideration as warm dark matter in the range 0.001–0.05 MeV/c^2. Cold dark matter is the most popular candidate for the bulk of dark matter and corresponds to the WIMPs (Weakly Interacting Massive Particles) with a much higher mass energy range of 10^4 - 10^7 MeV/c^2. For a particle to be cold (slow-moving), it usually must be massive enough to have become non-relativistic early. Focusing on the very light boson particle under consideration as a sterile neutrino remains intriguing as a lower mass sterile neutrino candidate as a projection from the neutrino family generations, though it would be a “rebel neutrino” by most prominent opinions by being lighter than the electron neutrino. To all such considerations, there is probably a lower mass energy limit of about 45 eV/c^2. For some kind of sterile neutrino, that would imply a significant free-streaming length, suppressing small-scale structure, now on a scale in potential conflict with cosmic structure formation data. That is, such a particle can travel without interacting significantly before getting trapped or slowed by gravity or other forces. In cosmology, it is a measure of how far particles of dark matter or neutrinos can travel in the early universe due to their thermal velocities. While these points need much clarification, the range -8 down to -10 on the generation number 0 axis in Fig. 1 possibly represents a minimum particle mass, and possibly a candidate for a particle of dark matter as a "very light boson". Some theories do predict or allow smaller masses. As discussed in Section 10, axion-like particles (ALPs) deemed of cosmological significance are plausible occupants of the range -7 down to -11 on the generation number 0 axis, and so reasonable candidates for the linear log intercept of -8 MeV/mc^2 being relevant to dark matter.

An important aspect of Fig. 1 that leads to the above considerations is the linear logarithmic relationship between the mass energies and generation numbers. It rises *circa* 4 log-MeV per electron series generations. This is intriguing if rest mass energies are seen as quantum states and generation numbers are no longer just labels, because logarithmic trends in energy levels are relatively rare, more typically applying to "toy" models in which, for example, artificial, often rather unrealistic potential energy functions are used to explore various aspects, typically of mathematical interest. If the potential is logarithmic then linear logarithmic plots can be obtained, or in the case of Dirac fermions with fractal dimensions. However, with the present interest in curved space, it is notable that they also arise in the context of quantum field theory in curved spacetime. Caution is required, however, because logarithmic intervals between energy levels can also appear in dimensional analysis or scaling arguments. As in Fig. 1, one initially takes logarithms of "dimensionful" quantities, based on human-contrived units of measurement, as opposed to being finally dimensionless such as the action expressed in reduced Plank units. For example, if one plots mass m in kilograms and then switches to grams, y = log(1000m) shifts the plot vertically up the "y axis" by log(1000) = 6.91 even though the physics has not changed. It does not affect the relative values from generation number to generation number, noting that similar arguments apply to the neutrino family for which relative values based on neutrino oscillations are more reliable than absolute values. There are situations in which unit choices can cause more severe problems, but when the present masses are expressed in Planck masses, the picture provided by Fig. 1 is not materially changed. The bottom quark and tau electron represent a 2-member cluster of generation number 3 with a mean mass-energy of roughly 5950 MeV as opposed to 1 MeV for generation number 1. Correspondingly, the strange quark and muon electron with generation number 2 have a mean mass energy of approximately 100. The generation 3 cluster has a mass of *circa* $5 * 10^{-19}$ Plancks, the generation number 2 cluster has an energy of $8 * 10^{-21}$ Plancks and generation 1 of *circa* $8 * 10^{-23}$. The increment from $\log(8 * 10^{-23})$ to $\log(8 * 10^{-21})$ is 4.6 and the increment from $\log(8 * 10^{-21})$ to $5 * 10^{-19}$ is 4.1 which are approximately linear and comparable to the corresponding increments of 4 and 4 as a best fit in Fig. 1. Similar comments apply to mainstreams I and III, given the dispersion of masses in each cluster. It remains that if the mainstreams are not truly logarithmic but better described by some other mass-dependent function, then for any reasonable choice of such function, the mainstreams I, I, II, and III would lie on continuous monotonic curves that still need to be explained, and likely with a convergence to an intersection that still needs to be explained.

While detailed models may represent dualizable descriptions that are essentially different views of the same physics, the models might be separated by considerations of factors such as scale. The notion of curled-up dimensions is reminiscent of string theory which considers very small scales of "curling", on the order of Planck lengths [10], but it could also apply in a manner somewhat analogous to the path of a photon in the so-called photon sphere at the event horizon of a black hole [11]. While such orbits around black holes have been argued to be unstable, that instability may not apply on smaller scales or orbits with quantization [11]. However, if those scales emerge from a study such as the present one with a mass value very roughly comparable with an electron, the above "trace mass energy" of 0.020 MeV corresponds to the vicinity of the ultraviolet region, a value commonly involved in electron transitions in atoms involving interaction with photons and is on atomic scale. In that case the effects of such dimensions would have been expected to have be seen experimentally, in contrast with the strings of string theory which are on a much smaller scale.

5 Theoretical Interpretations: Phase and Mass Separation

The use of electron volts as measures of mass [8] naturally arises from methods that can only deduce masses indirectly, through the scattering. Recall that in scattering theory, when a particle interacts with potential, it can be reflected or transmitted. The probability amplitudes for these processes can change, and the changes can be described in terms of phase shifts. Unitary transformations preserve the norm, so the change in the logarithm of the amplitude for a fixed time of exchange between two adjoint states corresponds to a phase shift, an "equal increment" that is a multiple of the imaginary part of the logarithm, as it represents the phase angle.

It is useful to separate the magnitude from the phase components of the wave function, and this is conveniently done in the Wentzel-Kramers-Brillouin (WKB) [12-14] approximation, a method related to the Liouville–Green method for finding approximate solutions to linear differential equations, particularly used in quantum mechanics. It treats the wave function as an exponential function with slowly varying amplitude and phase (compared to the de

Broglie wavelength), allowing for semi-classical treatment where some aspects are treated classically while others are treated quantum mechanically. WKB provides a means of relating the approximate density of states in the energy spectrum, dn/dE (see discussion of energy level index n below). In effect, one can validly imagine a wave encoding the properties at a particular point moving across a space occupied by a relatively smooth potential. That is, the potential U→U(x) in the orbital picture considered above. Notably, potentials discussed below are flat from the outset except for circular constraint. In the WKB approximation,

$$\psi(x,t) = A(x,t)e^{i(kx-\omega t+\phi)} \tag{1}$$

A is the amplitude and ϕ is the separated phase contribution. This separation seems appropriate for the situation that may apply in terms of photons in circular curled up orbits or as wave packets joined head to tail discussed in Section 7. As in Section 2, k = $2\pi/\lambda$ is the wave number and ω is the angular frequency, x is the position, applicable to both a classical extended spacetime dimension and a curled-up circular dimension. This would be relevant to Eqn. 2, as a description of each generation was one with basis states in equal superpositions corresponding to an increment in the phase ϕ = (1+g)π/2 for each generation number g arising from underlying spin-like states s^0, $s^{1,}$ s^2 and s^3, discussed next in Section 6, as appropriate. The amplitude A is the absolute value of ψ_0, i.e. $|\psi_0|$, which when squared to $|\psi_0|^2$ gives the exponential of mutual information 2(kx -ωt) as the action already expressed in units of reduced Planck's constant, e.g. k = $2\pi/\lambda$ = p/$\hbar$ where p is momentum and $\hbar$ is the reduced Planck's constant h/2π. Note that in applications of Schrödinger's equation to a system like a circular orbit, polar coordinates are suitable, ψ (r, θ, ϕ) which can be separated into radial R(r) and angular phase components Y(θ, ϕ), but without any length scale separation of the kind of the magnitude r is not the magnitude: it remains that he absolute magnitude is $|\psi(x, t)|$. In the case of separation, however, the potential U→U(x) and the solution of the Schrödinger equation is of the following simple form.

$$\psi(x) = A(x)e^{\mathrm{ikx}} \tag{2}$$

6 Theoretical Interpretations (The Number of Possible Generations as Determined by Phase)

That the above leads to consideration of 4 generation numbers 0, 1, 2, 3, and that 4 opens the consideration that the confinement to 4 generations, normally perceived as confinement to 3, might be explained by quarter phase shifts of $\pi/2$ that can occur in many physical and important mathematical systems. A potential that supports two bound states can lead to a total phase shift of 2π as 4 quarter-phase shifts. The Hilbert transform introduces a $\pi/2$ phase shift to all positive frequency components, and -$\pi/2$ to negative ones. It is of course the case that in trying to apply a phase model to the traditional 3 generation numbers, a phase interpretation already appears in systems with cyclic symmetry of order 3, or where wave phases are otherwise split into thirds of a full oscillation, does arise in various relevant systems. Notably, for the quarks, color charge transformations involve the SU(3) group, which includes phase relationships between three "colors", and some gauge transformations can involve $2\pi/3$ phase differences between components. Nonetheless, systems that are physical and involve quarter phase shifts appear to be more common than those involving thirds and tend to show some direct or close relevance to the class of models of potential interest here, e.g., 1-dimensional quantum scattering (the study of how a wavefunction behaves when it encounters a potential barrier or well along a single spatial dimension), quantum harmonic oscillators, but also more elaborate topological phases, as well as in path integral aspects discussed below.

The neutrino family may give important clues. Phase issues arise in the inclusion of two distinct potential clusters lying on the asserted generation zero axis in Fig. 1, giving 11 clusters as packages overall. In string theory and M-theory, spacetime is frequently proposed to have 11 dimensions, rather than the 4 (3 spatial and 1 time) of everyday experience, the further 7 being "curled up" in some manner. However, the track taken here is slightly different from string theory, examining both possibilities that the 11 clusters correspond to energy states in mass-energy spectra from one curled up dimension, or plausibly 3 sets of mass energy spectra represented by mainstreams I, II, and III. Mainstream III now becomes of interest. While absolute values are unknown, neutrino mass differences (squared) are known from oscillation experiments. If the neutrino masses m_1, m_2, m_3 obey a geometric progression, then $m_1 : m_2 : m_3 = r : r^2 : r^3$, and similarly from Fig. 1 and for particles generation number 1, one might reasonably surmise the following.

$$m_0 : m_1 : m_2 : m_3 = r^0 : r^1 : r^2 : r^3 = 1 : r : r^2 : r^3 \tag{3}$$

It is tempting to first consider the other families (series) of particles related to mainstreams I and II to be rooted in the notion of neutrino oscillations, which is the only example that nature clearly provides of spontaneous transition of one generation to another. More precisely, the generations are entangled with different weights dependent on time-of-flight. For a neutral neutrino pair seen as comprising M and its antiparticle as M*, then the entangled wave function can be considered from starting entanglement $|\,\psi(t{=}0)> = (1/\sqrt{2})(|MM^*> - |\,M^*M>)$. However, evolution of the wave function over time t requires expressing the wave function as a pure superposition of energy eigenstates $u|M> + v|M^*>$ discussed shortly below. More is known about the mass ratios between neutrino generations than the absolute values of their masses, so the notion of some kind of ground state on which the generations build log-mass increments is important. This is considered a matter of amplitude. Unfortunately, all that can be stated currently is that Eqn. 3 would be consistent with a linear plot of logarithms of energies versus generation numbers in the sense implied in Fig. 1. This tentatively suggests that each next generation of any fundamental particle, or as appropriate to the present paper, its "package", somehow involves an increase in dimensionality, say of the curled-up dimensions. From the slope of natural log 4, r ≈ 55 MeV for mainstream II and maximal possible masses of mainstream III, and roughly 149 Mev for mainstream I, it is noteworthy that $r^2 \approx 149^2$ for mainstream I corresponds very roughly to $r^3 \approx 55^3$ of mainstream II, suggesting possible involvement of an extra dimension for mainstream, but the simplest embodiments of that particular idea would seem to be in conflict with thinking regarding the convergence of I and II at generation zero discussed in Section 4.

The relevance of quarter phase shifts now enters as follows. One interpretation is that each step from generation-to generation-number involves the following kind of transformation of the basis state, based on the qubit, reminiscent of the neutrino oscillations. It is equally interpretable as spin. Consider the general description of the superposition between two basis states in terms of phase ϕ.

$$|\psi> = (1/\sqrt{2})(|0> + e^{i\phi}|1>) \tag{4}$$

For $\phi = \pi/2$,

$$|\psi> = (1/\sqrt{2})(|0> + i|1>) \tag{5}$$

This has a possible spin interpretation: the Pauli matrices generate rotations on spin-½ systems and a rotation by $\pi/2$ around one axis transforms basis states into equal superpositions. In spin systems or photon polarization, a $\pi/2$ phase difference creates states like circular polarization or spin eigenstates along different axes. For example,

$$|right\rangle = (1/\sqrt{2})(|horizontal\rangle + i|vertical\rangle) \tag{6}$$

That is, each generation is a spin eigenstate s, each generated from a spin transition from its predecessor, by a process related to effect of the Pauli spin matrix **S** as operator. Note that the effect is a combination of the states, which are not necessarily associated with the usual particle meaning of spin, but nonetheless the Pauli spin operator.

One may envisage an orthogonal rotation by i to the appropriate spatial axis so in this case imagining multiplication by i suffices as opposed to the use of spin matrices. That is, we may also replace the notion of Pauli spin operator by progressive multiplication by the imaginary number i. The 4 states for the 4 generation numbers 0, 1, 2 and 4 are then as follows.

$$s^0 = |0\rangle + i|1\rangle \tag{7}$$

$$s^1 = i|0\rangle - |1\rangle = is^0 \tag{8}$$

$$s^2 = -|0\rangle - i|1\rangle = is^1 \tag{9}$$

$$s^3 = -i|0\rangle + i|1\rangle = is^2 \tag{10}$$

and note that, of course,

$$s^4 = |0\rangle + i|1\rangle = is^3 = s^0 \tag{11}$$

Some additional comments should be made on the relation between this i-rotation and various relevant matrix operations. The quarter phase shift is an essential feature in path integral theory and interference phenomena. A maximum phase shift of $\pi/2$ occurs when transitioning between orthogonal quadrature components (such as position and momentum), and between different saddle-point contributions in semiclassical path integrals. In signal theory, orthogonal quadrature components refer to two signals that are 90 degrees out of phase with each other, allowing for the representation of a complex signal as two separate, orthogonal components. These components are often referred to as in-phase (I) and quadrature (Q) components; they are crucial in various signal processing and communication systems and provide a link between quantum and information theories. Important here is that one must consider a creation operator to address the next energy level in a spectrum that creates s^g from s^{g-1}, say called **S** to indicate some form of relationship with spin. Paul spin operators σ_x, σ_y, and σ_z each square to the identity operator **I**, so if adopting that kind of spin perspective, one should think in terms of **S** as the square root of a Pauli spin operator. While this square root exists, it is known to be neither unique nor Hermitian. It is of incidental interest that $\sigma_x \sigma_y \sigma_z = i\,\mathbf{I}$ but, along with notion of a Lorentz-like rotation around axes, this motivates thinking in terms of a spin operator as **S** as simply analogous to progressive multiplication i (see Eqns. 7-11) one may progress through Eqns. 5-9, and Eqn. 3 can be re-rendered as follows

$$m_0 : m_1 : m_2 : m_3 {}_{,} s^0 : s^1 = \mathbf{S}s^0 : s^2 = \mathbf{S}s^1 = \mathbf{SS}s^0 : s^3 = \mathbf{S}s^2 = \mathbf{SS}s^1 = \mathbf{SSS}s^0, \mathbf{SSSS}s^0 = s^0 \tag{12}$$

Note that for each generation g all have the same absolute magnitude squared of $| s^g|^2 = 1^2+1^2 = 4$, so these are purely phase contributions. The initial (equivalent to fifth) state with generation number 0, -4 log MeV in Fig. 1, i.e. the generation zero state on which **S** acts, may represent a photon as Fig. 1 asserts, but seems unlikely to be a free running photon packet in a curled-up circular dimension that would be a larger scale, at least the scale of electron-in-atom relations for which one may have expected to detect the impact experimentally, already. These aspects raise the question of what determines the amplitude in Eqns. 1 and 2, attempts to quantify it being as follows.

7 Theoretical Interpretations (Amplitude and The Linear Log Mass-Energy)

Here, some general observations, particularly in relation to the more conjectural qualitative proposals in Fig. 1, are made to give them some justification. Algebraic models for quantifying amplitude that are consistent with the linear logarithmic relation are discussed later in Section 8 and after. That is because estimation of the amplitude associated with each generation is more challenging than treatment of phase for several reasons. For example, as a candidate particle the notion in Fig. 1 that gluons are all zero generation cluster members with the neutrino, and as mentioned above, one conjecture was that a true lowest mass particle would be a generation zero particle that is potentially a physically existing colorless gluon. More plausibly it might be seen as a low mass boson marginally possible as a hot dark matter candidate, all as discussed in Section 4. The reason for first considering a colorless gluon is that the gluons are combinations of color and anti-color and the SU(3) group has 9 possible combinations of color–anti-color but only 8 of them form the adjoint (octet) representation accounting for the physical gluons. Against considering the colorless gluon are that the remaining colorless gluon does not transform under SU(3) and is most generally considered excluded by internal consistency in Quantum Field Theory. Standard quantum chromodynamics excludes the singlet gluon because it does not fit the required gauge structure, has no observed role in current hadronic physics, and is would be effectively non-interacting in quantum chromodynamics theory. Not least, there is (as yet) no experimental evidence for such a particle.

Nonetheless, the less obvious placing of certain particles is not entirely arbitrary and that includes the notion of the colorless gluon as having some analogy (same “package”) as a very low mass boson. It cannot be entirely excluded. There can be some kind of generation zero as a pre-symmetry state, a more primitive, unbroken-symmetry boson existing before color symmetry emerged or before confinement occurred, a “generation zero” singlet gluon from an earlier stage in the universe or in a more unified gauge theory, is persuasive and reminiscent of the photon emerging from electroweak symmetry breaking, or gauge bosons mix in SU(5) or SO(10). That tempts proposing a fundamental role as a “proto-gluon” in the mass fabric of space-time, a hidden sector gauge boson implied in many grand unified, deep underlying particle such as in “emergent theories” describing how macroscopic properties and behaviors of a system arise from the interactions of its microscopic components, even though these macroscopic properties might not be directly predictable from the behavior of the individual components alone, and preon theories. If gluons are composite in a manner analogous to particles being formed from preons, then as noted above a ”generation zero” state might represent a proto-gluon from which color degrees of freedom became differentiated.

A consideration of amplitude underlying ”trace energy” for a particle placed on generation number, would be quantitatively and qualitatively different from that of where Fig. 1 locates the photon and graviton. As introduced in Section 4, the above 0.020 log MeV appears to be interpretable as some energy equivalent as a ”trace” rest mass, though not necessarily a photon, which was a conjecture. The concept as the zeroth generation as associated with the photon must be used cautiously, because the intersection (off diagram) of the potentially massless neutrino series with the generation axis lies *circa* 4 log MeV/c^2 below that "trace" mass. Although there is no direct coupling of the photon with the Higgs boson, the Higgs boson H can decay into two photons ($H \rightarrow \gamma\gamma$), but this occurs via loop diagrams involving charged particles like the top quark or W boson, all of which being along with the Higgs in the topmost cluster in Fig. 1. In a loop diagram, particles travel in closed loops of Feynman diagrams, indicating that the interaction occurs through virtual particles, not directly observable but frequently appearing as a quantum correction accounting for effects like quantum fluctuations involving virtual particles briefly popping in and out of existence [1-3]. In other words, the Higgs first couples to a heavy particle (like a W boson or top quark), which "forms a loop", emitting the two photons. Since the photons must share the energy equally (in the Higgs rest frame), each has an energy corresponding to half the mass equivalent energy of the Higgs boson, which is approximately 62,500 MeV, natural log 11.04, now in the high-energy gamma-ray region of the electromagnetic spectrum, and on much smaller wavelengths than atomic scale.

There are constraints on the amplitudes that one might reasonably have, from around the mass of the Higgs down at least as far as the extension of the neutrino series mainstream III. Recall that an amplitude component as realized in Fig. 1 as the mass energy of a particle speculated as belonging to a generation number 0 of the neutrino and gluon mainstream. Possible candidates included a very light boson, a warm dark matter candidate, colorless gluon, or some revised, exotic kind of sterile neutrino, and so on. See Section 10 for more options and discussion. Any candidate there would be, as noted above and Fig. 1 suggests, be much lower than log (0.020) $\approx$ -4 MeV/c^2, the extrapolation of mainstreams I and II. The intercept of mainstream III generations 1-3 with the generation zero axis was at *circa* log(0.000335) $\approx$ -8 MeV/c^2. Mass energies too much higher would start to have impacts expected to be seen experimentally. Also, it is probably not appropriate to consider mass energies too much lower. Hot dark matter is associated with lighter particles than cold dark matter. While some argument was made above for particles at that point being a marginal candidate for dark matter, it would be warm or hot dark matter with particles approaching the speed of light. It is even less easy to argue 0.000335 MeV/c^2 as consistent with some kind of entity being a fundamental feature of mass-space-time but, if so, that would allow a lower range of amplitudes down to, presumably, the energy associated with vacuum fluctuations (or "quantum foam"). That mass energy depends on how one defines and measures it, as it is not a fixed value like a particle mass, but a scale-dependent concept tied to the vacuum energy density of quantum fields and general relativity. Based on the cosmological constant and dark energy mass scale variously estimated at about $2 \times 10^{-9} MeV/c^2$, expressing it in mass energy terms would it intercept the generation zero axis of Fig. 1 at -20.

The contribution to an amplitude seen as | s^g|^2 = 4 in normalization, has not so far been considered. While a matter of normalization, it remains that one can multiply |0> and |1> by a weight w to give amplitude A = w s^g and a probability | w s^g |^2 = w^2 to provide any value of amplitude. This is simply arithmetic and does not provide an explanation or insight. Any useful notion of amplitude will be impacted by the appropriate potential energy function for the model. While a particle in circular orbit would have kinetic energy and momentum and/or angular momentum, the obvious natural choice, or simplest starting point, is that the potential U is uniform along the circumference of the orbit as a distance x, i.e. U $\rightarrow$ U(x). For reasonably large orbits this is important, and the flat potential would justify the use of the WKB approximation discussed in Section 5.

Some information-theoretic observations may be made because they make the consideration of quantitative contributions to amplitude clearer. While the contribution of phase to generation number is a matter of bits and qubits, it is of interest to relate the contributions to rest mass energy in terms of information measures (natural units are used in this text, 1 bit gives $\log_e(2) \approx 0.6931$ nats). In some models (including models relevant here), amplitude A is associated with normalization and a probability *prior* to measurement, here P(x). More widely, though, this is not the case, and using only the prior would be more like confining attention to the partition function. The probability amplitude of e.g. <A | B> encodes a probability dual {P(A | B), P(B | A)}, i.e. the Dirac observable probabilities [4]. Most often the concept is of fixing or preparing B in <A | B> to measure A with respect to it (i.e. conditional upon B) to obtain P(A | B), and fixing A in <A | B> and measuring B in respect to it (i.e. conditional upon A). This is not necessarily obvious when using the Dirac recipe for observable probabilities [4], but it is entirely consistent with it. Here, one calculates P(A | B) from <A | B> starting from the ket normalization of <A | B> to give, say, <A | B>′ and then applying the Born Rule using $|<A \mid B>'|^2$. Somewhat similarly, one obtains P(B | A) from the bra normalization of <A | B> as ′<A | B> or more commonly by the ket normalization of <A | B> after complex conjugation <A | B>* = <B | A>, then applying the Born rule as $|<B \mid A>|^2$. This is well exemplified in the following model.

8 Amplitude and Chester's Introductory Bead-on-a-Circular-Wire Model

Chester's bead-on-a-circular-wire model [15] is a "toy" system that was intended as a gentle introduction in his Primer of Quantum Mechanics, but it becomes an appropriate model for considering the wave function associated with a hypothetical underlying entity in 1-dimensional circular space. The mass attributed to this entity is not the final rest mass which is in the present case seen as the energies $E = mc^2$ from the energy spectrum arising from the above wave function. The de Broglie mass associated with the wave confined to the circle is, nonetheless, a plausible candidate for m_{trace}. In Chester's model, however, the wave function spans the circumference and completes with continuity on the circle to give the energy spectrum of interest, implying that the scale being considered is small, and that of the wave packet. This is not necessarily the case in Section 8, where the wave packet does not span the circumference circle.

Considering only the single exponential e $^{i\alpha}$ where a is the action in reduced Plank units, then the action relates to the mutual information in natural units, say between x and y, as revealed by measurement. $|\psi_0|^2$ is then seen as giving the exponential of mutual information $\log(2\alpha) = 2(kx - \omega t)$, an "association probability" is given as P(x; t) = P(x, t) / P(x)P(t), more properly called the *association constant* K(x; t) of the exponential of mutual information I(x; t) = 2α. Chester's model for which his pure momentum state function which he writes as $\psi_p(x)$ = <x| p> has the *prior* probability P(x) = 1/L (meaning prior to measurement of location x) where L is the length of the wire (orbit).

$$\psi_p(x) = <x|p> = (1/\sqrt{L})e^{ikx} \tag{13}$$

See Chester's equation 2.8 for the bead-on-a-circular-wire model in Ref [15]. Note that by comparison with Eqn. 2, A(x) = $\sqrt{P}$(x) = $(1/\sqrt{L})$ for a uniform potential on a circular track. However, this seems misleading because for Chester the wave for the bead-on-a-circular-wire spans the track joined and joined head to tail in the manner of a closed string in string theory. In that case the continuity of the wave function requires the following constraint on the momentum.

$$p = nh/L, n = \ldots -2, -1, 0, +1, +2, \ldots \tag{14}$$

It is the integral $\int_0^L |<x \mid p = nh/L|^2\, dx = 1$ that leads Chester to the choice $(1/\sqrt{L})$ as the normalization term in Eqn. 13, but that remains the prior probability that the bead could be

anywhere on the track prior to measurement. Unlike the phase considered above, these are discrete energy levels that are not constrained to a specific number of states.

Note that the exponent as action in units of $\hbar = h/2\pi$ is $2\pi px/h$ but with x now as a distance x around a wire of length L in the bead-on-a-circular-wire model, px = nh(x/L) and the action in units of h is 2πnh(x/L)/h giving 2πn(x/L). It may be recalled that, in Eqn. 11, $k = 2\pi/\lambda$ the constant k identifies with the wave number, hence λ = L/nx, which provides no sense of the scale of x and L. Note that the interpretation of mass, energy, λ etc. varies somewhat with the model being considered, including Chester's model repurposed, and notably, for example, the rest mass energy as $E = mc^2$ is to be interpreted as the kinetic energy of a wave packet or continuous wave moving in a circular orbit. At first glance then, Eqn. 14 does not suggest the linear log rest mass energy of Fig. 1 and no reasonable manipulation gives the log energy form. The inverse form of the de Broglie relationship, m = h/λv, and assuming rest mass energy as the energy of a photon moving in a circular orbit at light speed v = c, then $mc^2 = hc/\lambda$, with λ = L/x, but rest mass energy seen as generated that way would still be linear in n =. . . -2, -1, 0, +1, +2,. . . However, Chester's model had a different purpose in which the bead is a true particle with mass m. For the n=0 level, exponential of the action nh(x/L) = 0 is simply 1, giving ψ(x) = A(x) = 1 and in Chester's model, represents zero ground energy. From the momentum spectrum Eqn. 14, the spectrum of the overall energy can be computed, and the resulting eigenvalues become more interesting. In the calculation of the quantum spectrum of the overall energies [15] in the model goes as the square of n.

$$E_{\mathrm{n}} = n^2h^2/2mL^2 \tag{15}$$

$$log_{\mathrm{e}}(E_{\mathrm{n}}) = 2log_{\mathrm{e}}nh/L\sqrt{2m}, n>0 \tag{16}$$

This model depends on the energy E_1, analogous to the generation number 1, not 0, giving the following energy spectrum.

$$E_0 = 0, E_1 = h^2/2mL^2, E_2 = {E_1}^2, E_3 = {E_1}^4, E_4 = {E_1}^9, E_4 = {E_1}^9, \ldots \tag{17}$$

The role of m in adapting this model to present purposes would then be one of considering the m as some kind of trace mass or equivalent, notably h/λv or $mc^2 = hc/\lambda$ according to model. This would not produce a linear plot of the logarithm of energy as a formal physical-mathematical solution, but given that Section 6 constrained solutions to 4 generations, and that generation number 1 particles are sufficiently close to form a cluster of circa $\log_e(E_1+E_{trace}) \approx 1$, where the E are the energies in mass, so that E_1+E_{trace} = 2.72 MeV/c^2 and (ii) $\log_e(E_{trace}) \approx$-0.020 MeV/$c^2$ so that $E_{trace} \approx$ 0.98 MeV, then one may write as follows.

$$log_{\mathrm{e}}(E_0) = log_{\mathrm{e}}(E_{\mathrm{trace}}), log_{\mathrm{e}}(E_1 + E_{\mathrm{trace}}), 2log_{\mathrm{e}}(E_{1+}E_{\mathrm{trace}}), 4log_{\mathrm{e}}(E_{1+}E_{\mathrm{trace}}) \tag{18}$$

$E_1+E_{trace} \approx$ 2.72 MeV/c^2 and $E_{trace} \approx$ 0.98 requires that $E_1 \approx$ 1.74 MeV/c^2. This is plausible but does not quite nail down a specific model. Many processes in particle physics are in or around that latter 1.74 MeV/c^2 range. Though detailed discussion is beyond present scope, they include Beta decay (P-32, Sr-90), neutron emission (fission) with average $\sim$ 2 MeV/c^2, gamma rays from nuclei common in decay and deexcitation, reactor antineutrinos with peak around 2-3 MeV/c^2, deuteron binding energy 2.2 MeV/c^2, positron annihilation (kinetic energy), and so on. At present, the notion of a bead-on-a-circular-wire model cannot be eliminated, but as discussed above, suggests string-theory-like models on the atomic scale that should have been readily validated.

While the above Eqn. 18 leaves open the interpretation of two kinds of trace mass, and arguably three if the extrapolation of the neutrino series is considered from Fig. 1, it is tempting to consider that $E_1 = E_{trace}$. Unfortunately, this leads, for the electron series expressed in MeV, to approximately E_0 = 0.020 MeV/c^2, E_1 = 0.04 MeV/c^2, E_2 = 0.00016 MeV/c^2, and E_3 = 0.00000256 MeV/c^2, a very different relationship. This obliges consideration of the mainstream plots as dependent on 4 parameters impacted by mainstream identification I, II, III, though it must be recalled that any reference to mainstream III is a reference to upper limits on rest mass of members of the neutrinos at the time of writing. This potentially requires 2 more parameters to incorporate the neutrino series. The bead-on-a-circular-wire model is not a perfect fit or explanation for Fig. 1, and it is far from a complete theory, but it seems to have the right trend. However, other explanations need to be explored.

9 Wave Packets in Larger Curved Space: The Klein-Gordon Perspective

The orbital class of models in Section 2 was agnostic and democratic in terms of the origin of rest-mass energy being due to, e.g., photons moving in some kind of circle, reminiscent of the photon sphere around a black hole. In this Section, the de Broglie wavelength Chester bead, or the effective length of a photon wave packet as the product of its to its coherence time and the speed of light, are allowed to be small in comparison to the length of a curved space. Equations with masses m as even powers appear in certain physical descriptions and notably as describing behavior in 4-dimensional spacetime, i.e. a distance in the space with dimensions x, y, z, -*i*ct. The basic nature of the Klein-Gordon equation is as a wave equation analogous to the Schrödinger equation with wave function ψ with eigenvalues m^2 and hence derivable from the considerations above but now applied to the four dimensions of spacetime. It originally applied to all spinless particles with positive, negative, and zero charge, but has been generalized to include spin [17]. Its simplest form is in natural units ($\hbar$ = c = 1), as follows.

$$\Box\psi(x') = -m^2\psi(x') \tag{19}$$

Here $\Box$ is the d'Alembertian operator or wave operator $\Box_\gamma = (1/c^2)\,\partial^2/\partial t^2 - \nabla^2$ for a metric γ replacing the Laplace operator that deals with flat Minkowski space. Consistent with that, the x′ is valid for a distance x but is more properly, or generally, a distance x, y, z, or -*i*ct or a distance space define by two or more of these dimensions.

The equation is linear in m^2 not in log(m). However, the interest here is in a curved space such as a curled-up dimension or the vicinity of a black hole. In such a curved spacetime, the Klen-Gordon approach yields the following.

$$m^2\psi = \Box_\gamma\psi - \xi R\psi \tag{20}$$

This equation describes the relativistic propagation of a scalar field with mass m on a curved spacetime with metric γ again written using the d'Alembertian operator and a curvature described by R. Again, the solutions go as m^2. More explicitly, one may write as follows:

$$m^2\psi = \frac{\hbar^2}{\sqrt{(-\gamma c^2)}}\,\partial_\mu\Big(\gamma^{\mu\nu}\sqrt{(-\gamma)}\,\partial_\nu\psi\Big) \tag{21}$$

Here γ is the determinant of the metric tensor that replaces the Minkowski metric for flat spacetime with the general metric tensor. There was thus no notion of log(m) hidden in Eqn. 20. The Klein-Gordon equations can be generalized to include any potential V.

$$\Box\psi(x') = -\partial U/\partial\psi \tag{22}$$

In principle, a potential can be selected to generate a linear log relationship of mass energies. A common choice of potential U= U(ϕ) of potential which arises in interactions is as follows.

$$U(\phi) = 1/2 \cdot m^2\phi^2 + \lambda\phi^4 \tag{23}$$

This does not yet have the required property of log distribution of mass-energy states, but in quantum corrections to such equations in field theory (e.g., renormalization group equations or effective actions), terms like log(m^2) may appear. Theoretical treatments that at some point build on Klein Gordon equations with the required linear mass-energies can appear in loop corrections, quantum corrections to a physical quantity (like mass, charge, or a propagator) that arise from virtual particles appearing in Feynman diagrams with loops. Under specific spacetime geometries and conditions, the Klein-Gordon equation in curved space can lead to an approximately logarithmic spectrum of energy levels usually associated with cosmological scales, particularly in Anti-de Sitter (AdS) backgrounds, near black holes, or with certain boundary conditions. This behavior is not universal and depends on the geometry and potential structure, but in the AdS case, the Klein-Gordon equation has normal modes that often take the following form

$$log(\omega_{\mathrm{n}}) \approx log(n + \Delta) \tag{24}$$

The conformal dimension Δ depends on the mass and spacetime dimension, and in certain compactifications or radial quantizations, energy levels can appear logarithmically spaced. Near black holes (e.g., Schwarzschild, Kerr, AdS black holes), one can find quasinormal modes of scalar fields. The imaginary parts of such frequencies sometimes behave approximately logarithmically with the mode number.

$$Im(\omega_{\mathrm{n}}) \approx log(n) \tag{25}$$

On less than cosmological scales, in effective 1D systems with radial or box-like boundary conditions, such as finite, small-scale Anti-de Sitter slices, braneworld models, and fields in warped extra dimensions, one can engineer effective potentials where the mass energy m_n levels scale logarithmically.

$$m_{\mathrm{n}} \approx log(n) \tag{26}$$

This is sometimes used in holographic quantum chromodynamics to reproduce Regge trajectories. Again, however, these are somewhat artificial constructions rather than experimentally observed cases. Eqn. 25 does in contrast reflect what is believed the decay spectrum of the field around a black hole, though the final amplitude does not represent only the imaginary part of ω_n. Detailed discussion of all the above is beyond present scope, but see Ref [18] for sources. That paper is also of direct interest by investigating the Klein-Gordon equation in the past causal domain of a de Sitter brane embedded in an Anti-de Sitter bulk. Solving the global mixed hyperbolic problem related to conformal spaces in relativity. Any finite energy solution was shown to be expressible as a Kaluza-Klein tower that is a superposition of free fields in the Steady State Universe, of which Ref [18] studied the asymptotic behavior. Importantly, it shows that the leading term of a gravitational fluctuation is a "massless" graviton. This is, of course, to be to distinguished from a vacuum fluctuation, by many magnitudes, they are related concepts with related mathematics in Quantum Field

Theory. An optical vacuum fluctuation is of the order 10^{-6} MeV/c^2 to be compared with a gravitational wave (LIGO scale) of 10^{-19} MeV/c^2 while a Planck scale fluctuation in spacetime is of order 10^{21} MeV/c^2, the Planck mass. For Fig. 1, the first of these is dwarfed by the m_{trace} energy of 0.020 MeV/c^2, and it should be noted that while projecting the neutrino series gives an intercept with generation number 4 log MeV/c^2 units below the photon and ”massless” graviton, the gravitational wave would lie at an intercept point far below that.

10 Conclusions

This study aims only to discuss classes of model that might explain Fig. 1. At present, the model (Eqn. 18) based on Chester's simple introductory model to quantum mechanics may be the most promising explanation of a plot logarithmic of mass energy versus generation number, taken in the context of the discussion around it. It is not truly a log mass-energy model. Intuitively it seems to call into question the log linear extrapolation to particles with a generation number 0 less attractive, but by no means eliminated. It becomes more acceptable considering (a) the limitation to 4 generations and (b) differences between the masses within clusters, small differences in the present context, that are presumably due to perturbation by other influences such as vacuum fluctuations and polarization.

Recalling that, at several points in the text, it was noted that logarithmic mass energy relationships are rare but seen in systems with logarithmic or scale-invariant potentials, models with exponential degeneracies, engineered quantum systems, or certain black hole and conformal field theories. To the examples set there should be added models that at first seem less plausible. One is models of quark confinement, a notion that sounds promising, but the models only arise in some approximations. Again, many of these possibility seem more like exploratory or "toy" cases. It is the case that Klein-Gordon equations in curved space can give close-to-truer logarithmic distributions of energy states, but it is not completely clear how realistically physical these models are. There are also certain models of quantum chaos addressed by random matrix theory [16]. This latter is of interest for several challenging reasons. Apart from the postulated collapse of the wave function and arguably creation/annihilation operations, basic quantum mechanics is generally seen as a linear algebraic system focusing on the dynamical behavior of specified particles. This is, however, less so for Quantum Field Theory QFT that manages interactions and transformations between particles. QFT couples to standard quantum theory as Schrödinger wave mechanics through the form of Dirac dualization, discussed in Section 1.

The possible relationship between the intercept at about -8 of the linear log extension of mainstream III (seen as a neutrino series) with the generation number 0 axis and dark matter, albeit off the figure as being very hypothetical, was nonetheless considered interesting. Recall that sterile neutrinos are considered heavier than other members of the neutrino family, and it is the largest possible masses of neutrinos, by experiment, that are plotted in Fig. 1. The intercept at the generation zero axis is obviously lower than the electron neutrino. The idea of a colorless gluon occupying the role around that value of -8 on the generation number 0 axis would be far from popular in standard theory for several reasons beyond present scope, though SU(3) × U(1) extensions could admit extra bosons with different transformation properties and distinct physics. The intercept of *circa* -8 of log mass energy deducible from Fig. 1 does, however, overlap with estimates of mass energy ranges for a variety of other, primarily hypothetical or un-characterized particles, notably exotic neutrinos, axion-like particles, dark photons, the gravatino, and Kaluza-Klein states, and warm dark matter. Not all of these are reasonable candidates for the -8 intersection in their current model rm. For example, as discussed as candidates in Section 3, there exotic neutrinos called very light neutrinos (VLNs) are usually considered as having masses in the range 0.000001-0.00001 MeV/c^2, which would put them in the range -11 down to -14 on the generation number 0 axis, rather too low to occupy the -8 intercept. Of the above possibilities, however, only axion-like particles are believed to have zero spin, in parity with the photon and popular model of the graviton on the same zero generation axis. If the data mining and analytical studies in Section 2 are to conserve particle spin zero as a pseudoscalar bosons, odd parity (change sign under spatial inversion), then this gives a closest fit with axion-like particles (ALPs), low mass bosons consistent with broader, non-QCD models, still considered strong candidates for cold dark matter. On the relevant cosmological scale, ALPs are believed to have masses in the range of 10^{-6} - 10^{-3} MeV/c^2, about -7 down to -11 on the generation number 0 axis in Fig. 1, so making them reasonable candidates as generation number 0 members of mainstream III.

There remains a further and perhaps more dramatic explanation of Fig. 1. The apparent linear logarithm relationship combined with a notion that the action in reduced Planck units is in essence an information measure, and if expressible in terms of a finite number of bits in an appropriate model of increasing micro-states per generation number might provide a new insight. These aspects are at this stage unclear, but worthy of investigation.

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