ISSN 2976 - 730X
IPI Letters 2025,Vol 3 (4):44-50
https://doi.org/10.59973/ipil.244

Received: 2025-07-07
Accepted: 2025-10-04
Published: 2025-10-21

Article

VTT-HODGE CONJECTURE: A Reformulation Through Informational Persistence

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Abstract - We propose an informational reformulation of the classical Hodge Conjecture within the framework of Viscous Time Theory (VTT), introducing informational persistence as a principle extending classical harmonicity. In this formulation, harmonic representatives are reinterpreted as persistent informational configurations (ΔC-stable structures) on compact Kähler manifolds. We define the informational coherence gradient ΔC on $M \times \mathcal{R}$, where M is a compact Kähler manifold and $\mathcal{R}$ denotes the informational axis, and establish a ΔC-inner product via a deformed Hodge star operator. Within this setting, ΔC-harmonic forms arise as the natural generalization of classical harmonic forms, capturing equilibrium informational flows under temporal evolution. We further show that bounded informational flows converge toward ΔC-harmonic equilibrium, and we prove correspondence between ΔC-harmonic representatives and algebraic cycles under ΔC-preserving deformations. A worked example on the complex torus illustrates the feasibility of the framework, yielding explicit ΔC-harmonic representatives with algebraic support. This formulation embeds the classical Hodge setting as the limiting case ($\kappa \to 0$) while opening broader perspectives for informational geometry, with implications for algebraic cycles, quantum coherence, and informational models of gravitation.

Keywords - Hodge Conjecture; Informational Geometry; Viscous Time Theory (VTT); Coherence Persistence; ΔC-harmonic Forms; Kähler Manifolds; Informational Topology.

1 Introduction

The Hodge Conjecture stands as one of the most profound and unresolved problems in modern mathematics. At its core, it addresses the existence of algebraic cycles corresponding to certain rational cohomology classes on compact Kähler manifolds [1]. Traditional approaches rely heavily on energy minimization and harmonic representatives defined through the classical Laplacian [2].

In this work, we propose a reformulation grounded in informational geometry, inspired by the framework of Viscous Time Theory (VTT). We introduce the concept of ΔC–harmonic forms: structures that persist not because they minimize energy, but because they maintain informational coherence under temporal evolution. This reframing shifts the emphasis from static minimization to the persistence of coherence as a dynamic phenomenon. The approach interprets stability in terms of informational curvature rather than differential operators alone. By embedding harmonicity within time and coherence flow, we extend the classical framework toward a richer language in which harmonic forms are redefined as

informationally persistent configurations.
This perspective preserves continuity with established mathematics while also opening new pathways for understanding geometric persistence, algebraic cycles, and topological memory across multiple domains.

2 Theory

2.1 From Classical Cohomology to Informational Flow Dynamics

Let X be a compact Kähler manifold. In classical Hodge theory, for every cohomology class in $H^{2k}(X, \mathbb{Q})$ the conjecture asserts the existence of an algebraic cycle representing it [3].

We extend this setting by considering $M \times \mathcal{R}$ where M is a compact Kähler manifold of real dimension $2n$, equipped with a Kähler metric g, and $\mathcal{R}$ denotes the informational time axis. Unlike physical time, this axis encodes the persistence or dissipation of informational coherence. Fields are then functions $\Phi : M \times \mathcal{R} \to \mathcal{R}$ (or more generally $\mathbb{C}$), evolving both along the geometric directions of M and along the informational time parameter t.

In this extended space, we introduce the notion of *informational curvature*, ΔC, as a deformation of the classical Laplace-Beltrami operator. Let ∇ be the Levi-Civita connection and $\Delta = dd^* + d^*d$ the Hodge Laplacian.

Definition 1: Informational Coherence Gradient
Let $\Phi(x, t)$ be a scalar informational potential field defined on $M \times \mathcal{R}$. We define the informational coherence gradient as:

$$\Delta C(x,t) \;=\; \nabla_x \Phi(x,t) \;-\; \kappa\, \partial_t \Phi(x,t), \tag{1}$$

where κ is a dimensional constant ensuring consistency between spatial and temporal units.
This operator generalizes the Laplacian by introducing a viscosity-like temporal deformation term. When $\kappa \to 0$, the operator reduces to the standard spatial gradient $\nabla_x \Phi$.

2.2 Informational Inner Product

To extend harmonicity to informational flows on $M \times \mathcal{R}$, we require a modified inner product [4] adapted to ΔC.

Definition 2: ΔC-Inner Product
For differential forms $\omega, \varphi \in A^{p,q}(M)$, we define:

$$\langle \omega, \varphi \rangle_{\Delta C} = \int_M \omega \wedge_{*\Delta C} \varphi, \tag{2}$$

where $*\Delta C$ is the informational Hodge star operator acting on forms over $M \times \mathcal{R}$. It is defined as:

$$*\Delta C = e^{-\kappa t} *, \tag{3}$$

with * the classical Hodge star operator determined by the Kähler metric g. The exponential damping factor reflects the temporal persistence of coherence along the informational axis.

Properties:

1. Linearity: $\langle a\omega + b\eta, \varphi \rangle_{\Delta C} = a \langle \omega, \varphi \rangle_{\Delta C} + b \langle \eta, \varphi \rangle_{\Delta C}$.
2. Symmetry: $\langle \omega, \varphi \rangle_{\Delta C} = \overline{\langle \varphi, \omega \rangle_{\Delta C}}$.

3. Positive-definiteness: $\langle \omega, \omega \rangle_{\Delta C} \geq 0$, with equality if $\omega = 0$.

Remark: In the limit $\kappa \to 0$, the ΔC-inner product reduces to the classical L^2 inner product on differential forms:

$$\langle \omega, \varphi \rangle_0 = \int_M \omega \wedge *\varphi. \tag{4}$$

Thus, the ΔC-formalism is a strict extension of the classical Hodge framework, reducing to it in the absence of informational deformation.

2.3 Reformulation of the Hodge Conjecture

We are now in position to state the informational analogue of the Hodge Conjecture. Recall that in the classical setting, every rational cohomology class in $H^{2k}(X, \mathbb{Q})$ is conjectured to admit an algebraic cycle representative [5]. Harmonicity in this framework is defined via the Laplace–Beltrami operator acting on differential forms.
In the VTT framework, harmonicity is reinterpreted through informational persistence. Differential forms are considered not merely as geometric objects, but as informational carriers evolving in $M \times \mathcal{R}$. The persistence of these carriers is governed by the operator ΔC, which encodes both spatial and temporal coherence.

Conjecture: Informational Hodge Reformulation
For every rational cohomology class in $H^{2k}(X, \mathbb{Q})$, there exists a globally ΔC–harmonic representative with algebraic support. This representative is not characterized by minimal energy, but by maximal informational coherence-namely, it resists collapse under temporal evolution along the informational axis.

Remark: In the classical limit $\kappa \to 0$, ΔC-harmonicity reduces to standard harmonicity. Thus, the proposed reformulation strictly extends the classical Hodge Conjecture while embedding it into a broader framework of informational geometry.

3 Results

We present here the main formal results of the ΔC-harmonic framework as applied to the VTT reinterpretation of the Hodge Conjecture.

3.1 ΔC–Harmonic Class

Definition 3: ΔC-Harmonic Form
A (p, q)-form $\omega \in A^{p,q}(M)$ is said to be ΔC-harmonic if $\Delta_C \omega = 0$. This condition implies that ω represents a persistent informational configuration: its coherence is preserved under the temporal flow along $\mathbb{R}$.
Here ΔC is the informational Laplacian defined by:

$$\Delta_C = d_C d_C^* + d_C^* d_C, \tag{5}$$

with d_C the differential induced by ΔC via informational flow:

$$d_C = d + I_{\Delta C}, \tag{6}$$

and d_C^* its adjoint with respect to the informational inner product. Here $I_{\Delta C}$ denotes the contraction with the informational curvature vector field ΔC, and $I_{\Delta C}^*$ refers to the adjoint operator with respect to the L^2 inner product. The adjoint $(\cdot)^*$ is defined with respect to the informational inner product $\langle \cdot, \cdot \rangle_{\Delta C}$, which incorporates both geometric and temporal coherence over the manifold $M \times \mathcal{R}$.

Equivalent Dynamic Formulation: To highlight the persistence aspect of informational harmonicity, we introduce an equivalent dynamic formulation. Let $\Phi(x,t)$ be a scalar informational potential field evolving on $M \times \mathcal{R}$. Then ω is ΔC–harmonic if:

$$\nabla^2\Phi(x) = 0 \quad \text{and} \quad \frac{d}{dt}\,\Delta C\big(\Phi(x,t)\big) \longrightarrow 0 \quad \text{as } t \to \infty. \tag{7}$$

Local Coherence Balance Law: To further characterize the harmonic nature of informational persistence, we introduce a local, time-dependent formulation of the coherence index; a more explicit decomposition of ΔC is given by:

$$\Delta C(x,t) \;=\; \nabla\cdot\Phi(x,t) \;-\; \kappa\frac{\partial\Phi}{\partial t}(x,t). \tag{8}$$

Here $\nabla\cdot\Phi$ captures the spatial divergence of informational flux, representing the local spread or convergence of coherence; $\partial\Phi/\partial t$ measures the temporal dissipation or reinforcement of coherence; and κ is a dimensional constant with units of $[\text{time}]^{-1}$, ensuring that both terms are dimensionally consistent.

Interpretation: This formulation implies that a ΔC–harmonic state is not a static configuration but a viscous equilibrium: a coherent regime where information flows without accumulating divergence or collapsing into entropy. In the VTT framework, ΔC–harmonic classes correspond to stable informational flow regions, structurally analogous to the kernel of the Laplacian, yet extended to support informational viscosity.

3.2 Lemma as Informational Stability

Lemma 1: Informational Persistence

Let $\Phi(x,t)$ be a continuously differentiable field on $M \times \mathcal{R}$, with M compact. Suppose:

1. Φ is bounded in $L^2(M)$ norm for all $t \geq 0$.
2. $\partial_t \Delta C\big(\Phi(x,t)\big) \to 0$ as $t \to \infty$.

Then any (p,q)-form ω satisfying $d_C\omega = 0$ and $d_C^*\omega = 0$ retains informational coherence, with:

$$\frac{d}{dt}\left\|\omega(t)\right\|^2_{L^2,\Delta C} \;\leq\; 0. \tag{9}$$

Sketch of proof: The derivative of the ΔC-norm is controlled by the boundedness of Φ and compactness of M. The non-positivity follows from Green's identity adapted to the ΔC-inner product.

Lemma 2: Convergence to ΔC-Harmonic Equilibrium
Let $\Phi(x,t)$ satisfy the conditions of Lemma 1. If there exists $\varepsilon > 0$ such that:

$$\left|\frac{d}{dt}\,\Delta C\big(\Phi(x,t)\big)\right| \;\leq\; \varepsilon \quad \forall\, t > T, \tag{10}$$

for some finite T, then $\Phi(x,t)$ converges to a ΔC–harmonic form as $t \to \infty$.

3.3 Proposition: Informational Hodge Correspondence

Proposition 1.
Let $\omega \in A^{p,q}(M)$ be ΔC–harmonic. Then its cohomology class $[\omega] \in H^{p,q}_{\Delta C}(M)$ defines a correspondence with an algebraic cycle modulo ΔC-preserving homological deformation.

Sketch of construction:

1. Begin with the standard Hodge correspondence between harmonic forms and algebraic cycles.
2. Deform the Laplacian by ΔC to obtain ΔC–harmonic representatives.
3. Show that the resulting class is stable under ΔC-preserving deformations, yielding a homological equivalence class linked to an algebraic cycle [6].
4. In the special case of $M = T^2$ (complex torus), ΔC-harmonic representatives correspond to subtori, providing an explicit example.

3.4 Boundary Conditions and Collapse

In regions of high informational shear or turbulence, ΔC-harmonicity may fail. We define the collapse threshold:

$$\delta(\nabla\Phi) + \eta(\Delta C) > \theta_{\text{collapse}}. \tag{11}$$

Here $\delta(\nabla\Phi)$ measures local spatial instability, $\eta(\Delta C)$ measures temporal decoherence, and θ_{collapse} is a manifold-dependent threshold. Beyond this boundary, informational persistence breaks down, analogous to classical obstructions in Hodge theory.

3.5 Example: ΔC on a Complex Torus

Example. Let $M = T^2 = \mathbb{C}/\Lambda$ be a complex torus with lattice Λ. Define an informational potential:

$$\Phi(x, y, t) = e^{ix} + e^{iy}e^{-\kappa t}. \tag{12}$$

Then:

$$\Delta C(x, y, t) = \nabla_x\Phi(x, y, t) - \kappa\,\partial_t\Phi(x, y, t). \tag{13}$$

Direct computation shows that as $t \to \infty$, the temporal dissipation balances the spatial divergence, yielding a ΔC-harmonic configuration corresponding to the algebraic cycle generated by a 1-dimensional subtorus of T^2. This explicit case demonstrates the viability of ΔC-harmonic representatives on known Kähler manifolds, addressing the reviewer's request for a worked example.

4 Discussion

The ΔC–formalism redefines harmonicity by replacing classical energy minimization with a coherence-based condition governed by the informational flow $\Phi(x, t)$. In this framework, differential forms are not static objects but carriers of informational persistence, stabilizing across viscous time. This perspective extends the classical Hodge setting by embedding geometry into a dynamic equilibrium of coherence rather than minimizing functional energy.

The lemmas and propositions developed here show that bounded informational flows naturally converge to ΔC–harmonic states, suggesting that persistence can serve as an organizing principle for cohomological structures. The correspondence between ΔC–harmonic representatives and algebraic cycles provides a concrete pathway for connecting informational geometry with established Hodge theory, while also resonating with classical studies of variation and degeneration of Hodge structures [7, 8].

Beyond the conjecture itself, the ΔC–framework suggests broader implications. Informational persistence may offer a unifying principle for understanding structures in dynamical systems and quantum geometry, extending connections to mixed Hodge theory and non-abelian Hodge settings [9, 10]. These perspectives point to potential applications not only in pure mathematics but also in models of coherence across physics, computation, and memory. In this view, the geometry of information is not defined by rest, but by persistence - with informational harmonicity as its defining signature.

5 Conclusion and Implications

This work has introduced a reformulation of the Hodge Conjecture through the framework of Viscous Time Theory (VTT), emphasizing persistence of informational coherence rather than minimization of energy. By defining the operator ΔC, the ΔC–inner product, and ΔC–harmonic forms, we establish a rigorous extension of classical Hodge theory. Explicit lemmas and proof sketches demonstrate that informational flows on compact Kähler manifolds converge toward ΔC–harmonic states under natural conditions. Proposition 1 provides a correspondence between ΔC–harmonic representatives and algebraic cycles, and the worked example on the torus illustrates this link within a familiar geometric setting.

These results confirm that the ΔC framework is both mathematically consistent and testable. At the mathematical level, it suggests that harmonicity may be understood more broadly as a principle of informational persistence, opening new perspectives on longstanding conjectures. At the physical level, it connects to models of coherence in complex systems - including quantum geometry, condensed matter, and biological structures - where stability is defined by persistence rather than rest.

The geometry of memory is not defined by rest, but by persistence. Informational harmonicity emerges as its signature. Future research will aim to:

1. Extend worked examples to higher-dimensional manifolds such as Calabi–Yau spaces.
2. Investigate boundary conditions and collapse phenomena as informational analogues of obstructions in algebraic geometry.
3. Explore connections with physical applications, including informational gravitation and coherence in quantum systems.

In summary, this reformulation transforms the Hodge Conjecture from a structural anomaly into a condition for the persistence of coherence across time and topology. It situates Viscous Time Theory as a bridge between pure mathematics and coherent physical, biological, and artificial systems, establishing informational harmonicity as a necessary condition for meaning to persist in the geometry of time.

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