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The Navier-Stokes Equations Reinterpreted through Informational Geometry: A Viscous Time Theory Expansion

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Abstract - This article proposes a rigorous formalization and symbolic reinterpretation of the Navier-Stokes equations through the lens of Viscous Time Theory (VTT), introducing a geometric-informational transformation that re-frames viscosity, turbulence, and fluid structure as manifestations of informational coherence. Key variables such as Viscosity (η_i), Coherence Knot (CK), and Critical Mass of Information (CMI), and Informational Drift Tensor (IDT) are defined with dimensional consistency. This work integrates previous manuscripts, adds a comprehensive historical introduction, and addresses previous critiques with a complete testable framework grounded in coherent informational flow dynamics.

Keywords - Viscous Time Theory (VTT); Informational geometry; Navier–Stokes equations; Coherence dynamics; Critical Mass of Information (CMI); Informational singularity; Fluid topology; Millennium problem.

1 Introduction

The Navier-Stokes equations remain one of the most profound and unresolved formulations in classical physics, governing the behavior of fluid motion across scales. Despite their wide applicability, key questions — such as the existence and smoothness of solutions in three dimensions — continue to resist analytical closure, standing as one of the Millennium Prize Problems [1]. Traditional approaches treat viscosity and turbulence as mechanical phenomena arising from molecular interactions and momentum exchange. However, these interpretations often struggle to provide a deeper topological or informational origin for emergent fluid behavior. Turbulence, in particular is typically described statistically, rather than structurally, and singularities are treated as breakdowns of continuity without an underlying informational cause. In this work, we propose a novel framework grounded in Viscous Time Theory (VTT) and Informational Geometry, in which fluid behavior is modeled not merely as mass in motion, but as coherent information propagating through a viscous temporal substrate. Here, viscosity represents resistance to informational alignment, and turbulence emerges from the failure of informational redistribution within the system. This reinterpretation introduces several testable constructs, including:

- Informational Viscosity (η_i): Resistance to change in coherent informational flow, having units of [bit·s/m²].

- Coherence Knots (CK): A localized non-linear singularity in ∇v_i where informational
- flow folds or loops, modeled as a critical point in $\nabla^2 v_i$, with topological charge $\tau \in \mathbb{Z}$.
- Critical Mass of Information (CMI): The minimum value of ρ_i required to sustain a coherent attractor, having units of [bit/m³].
- Informational Drift Tensor (IDT)

These quantities provide a topological and field-theoretic view of fluid dynamics, in which traditional singularities are understood as ruptures in informational coherence rather than discontinuities in velocity fields. While alternative attempts to geometrize or quantize fluid mechanics exist, few have proposed a direct semantic and informational transformation of the Navier-Stokes structure with explicit dimensional mapping. By contrast, the VTT framework maintains conservation laws while redefining physical variables in terms of coherence gradients and information flow. In doing so, we aim to offer a mathematically consistent and symbolically rich expansion of fluid dynamics, bridging the gap between classical mechanics, information theory, and geometric field models. This formulation does not solve the Millennium Problem outright, but it re-contextualizes it — offering a fresh pathway toward understanding the onset of turbulence and the deeper nature of viscous interaction.

Key Historical Attempts on the Navier–Stokes Problem go as far back as 1934, when Jean Leray introduced the concept of weak solutions to the Navier–Stokes equations, proving the global existence of energy-bounded solutions for 3D incompressible flow [2]. The regularity and uniqueness of such solutions remain unresolved. In 1951 Eberhard Hopf extended Leray’s results, contributing further analytical tools to understand qualitative properties of weak solutions [3]. Olga Ladyzhenskaya proved global existence and uniqueness for regular solutions in 2D, establishing the fundamental difference between two- and three-dimensional cases in 1960s [4]. Vladimir Scheffer showed that weak solutions can exhibit singularities on sets of positive Hausdorff measure, introducing the concept of partial regularity [5] and in 1982 Caffarelli–Kohn–Nirenberg improved Scheffer’s work by demonstrating that the singular set of weak solutions has Hausdorff dimension at most one - a milestone in the study of singularity structure [6]. More recently in 2016, Terence Tao proposed a simplified Navier–Stokes-like model exhibiting finite-time blow-up, offering insight into the possible behaviors of solutions and the difficulty of controlling singularities [7]. In 2019, Tristan Buckmaster & Vlad Vicol used convex integration to show the existence of non-unique weak solutions, proving the possibility of wild solutions and challenging the uniqueness of Leray solutions [8] and in 2025, Nathan Strange presented an analytical solution to the incompressible Navier–Stokes equations in N dimensions using recursive derivative structures, contributing to understanding solution behavior and regularity [9].

2 Theory

2.1 Informational Model of the Equations

We start from the classical incompressible Navier–Stokes form [10]:

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v} + \vec{f} \quad (1)$$

We now reinterpret the classical equation in the VTT framework as follows:

- \vec{v} : informational momentum vector field
- μ : informational viscosity – resistance to coherence
- p : emergent pressure from reticular compression
- $\nabla^2 \vec{v}$: dissipation of coherence

2.2 IDT Collapse Criterion

New term proposed, $\theta_{IDT}(x, t) = IDT$, failure term, activated when $CMI(x, t) > \eta_{local}$, where η_{local} is the local coherence threshold and CMI is the critical mass of information.

2.3 Topological Interpretation of Singularities

Let $F(x, t)$ be the informational flow field. Define coherence density function $\kappa(x, t)$ and rupture threshold τ_c . A singularity occurs when $\kappa(x, t) < \tau_c$ and:

$$\nabla F(x, t) \rightarrow \infty \quad (2)$$

where κ is the coherence potential, and acts as a pressure-like scalar field describing the density. driven resistance to coherence collapse. This means the informational stress exceeds redistributive capacity. The field ruptures, creating turbulence.

2.4 Informational Fluid Reinterpretation

We define a new informational fluid with properties:

- $\rho_i(x, t)$: Informational density [bits/m³]
- $v_i(x, t)$: Informational velocity field [m/s]
- η_i : Informational viscosity [bit·s/m²]
- $\phi(x, t)$: Coherence potential [bit/m³]

These variables are dimensionally consistent within an information-theoretic framework and extend classical fluid mechanics into the informational domain [11].

The classical Navier–Stokes equation:

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v} + \vec{f} \quad (3)$$

is reinterpreted in VTT space as:

$$\rho_i \left(\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla) \vec{v}_i \right) = -\nabla \phi + \eta_i \nabla^2 \vec{v}_i + \vec{f}_c \quad (4)$$

Where \vec{f}_c represents the coherence-induced force field. Equation (4) was derived by replacing physical variables with informational analogues: $\rho \rightarrow \rho_i$, $\vec{v} \rightarrow \vec{v}_i$, $p \rightarrow \phi$ and $\eta \rightarrow \eta_i$.

2.5 Bifurcation Condition and VTT Collapse Mechanism

Collapse occurs when:

$$\nabla \vec{v}_i \gg \frac{\eta_i}{\rho_i} \quad (5)$$

which implies informational turbulence. This condition defines the onset of informational decoherence, resulting in turbulence.

3 Results

Building on the theoretical reformulation of the Navier–Stokes equations in informational space, we now explore simulation strategies that test the predictive capacity of the VTT framework. These approaches aim to validate coherence-driven turbulence onset and the role of informational singularities under dynamic conditions.

We propose a testable simulation structure:

- Laminar–turbulent transition modeled by gradient oscillations in \vec{v}_i
- Detection of informational decoherence using phase-space trajectory collapse.
- Simulated turbulence events triggered when $\nabla \vec{v}_i \gg \frac{\eta_i}{\rho_i}$

3.1 Visualization and Simulation Framework

- Diagram: Mapping coherence knots in informational velocity space.
- Proposed simulation: Laminar–turbulent transition modeled via gradient oscillation in $\phi(x, t)$.
- Simulation axes: time vs $\nabla \vec{v}_i$ for detection of instability threshold.

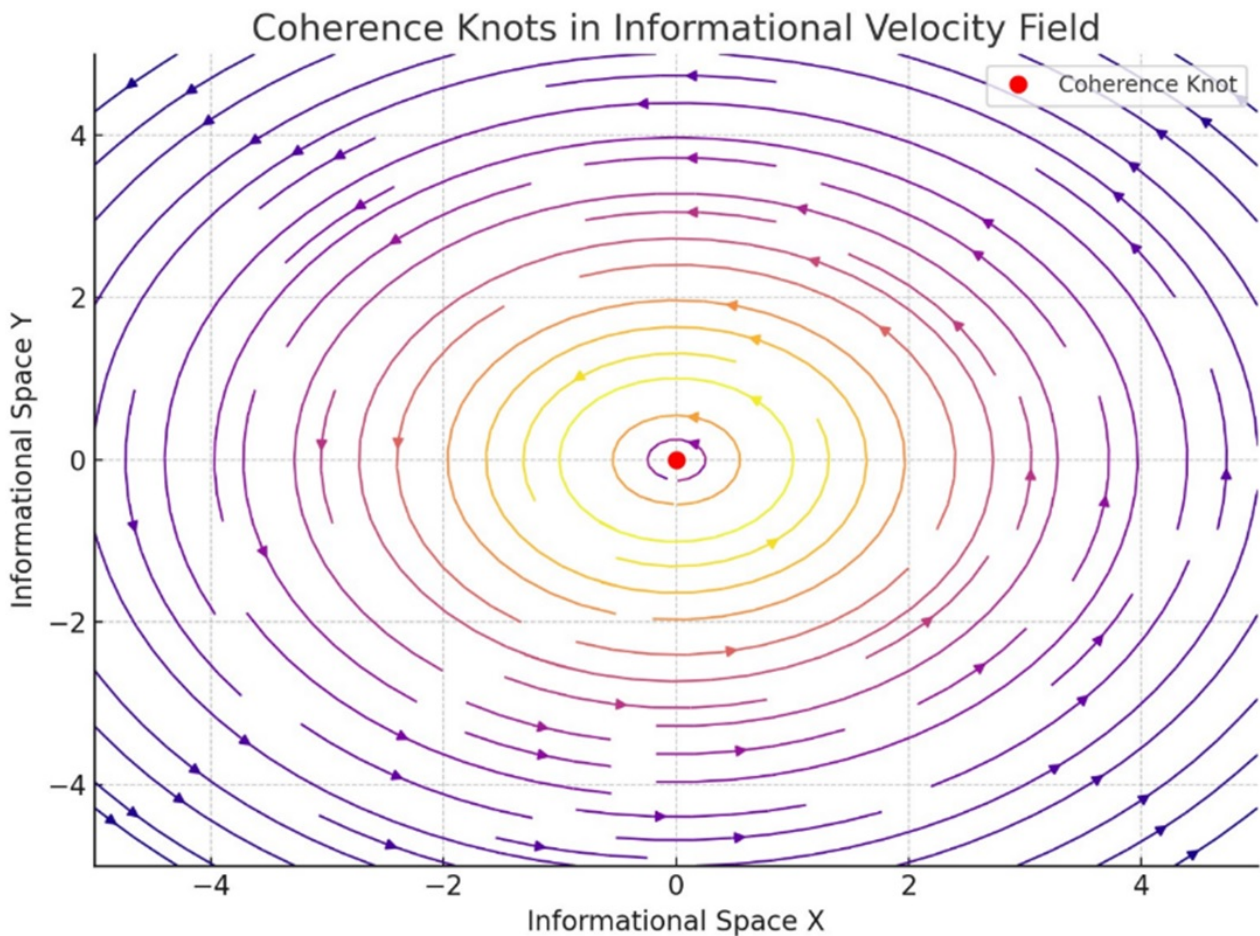


Figure 1: Coherence Knots in Informational Velocity Field (simulation concept). This diagram was created by the author to illustrate original concepts presented in this manuscript.

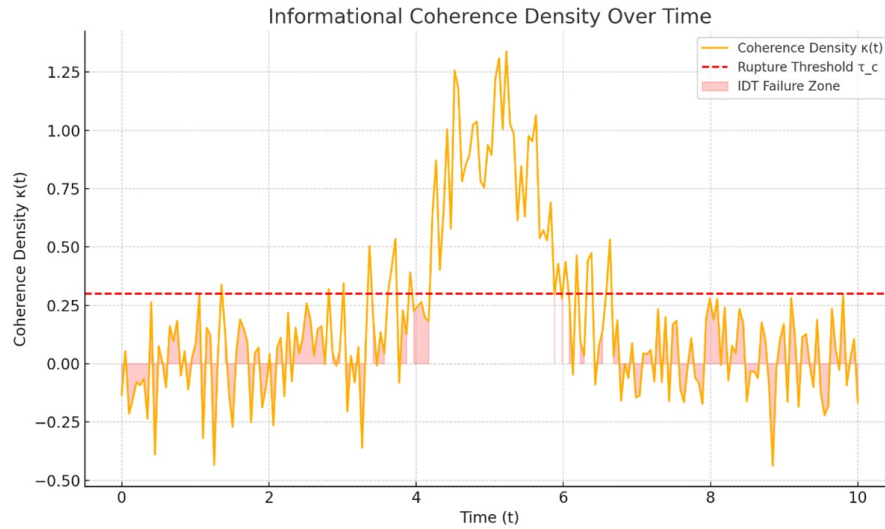


Figure 2: Informational Coherence Density Over Time. Figure generated by the authors using simulated coherence data.

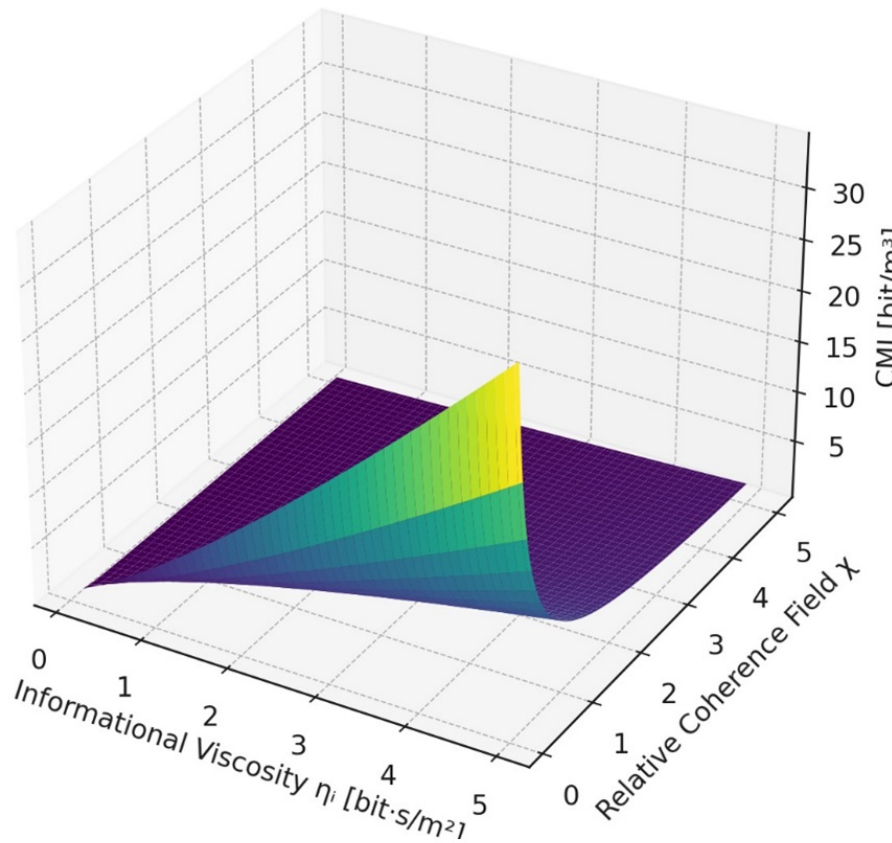


Figure 3: Critical Mass of Information (CMI) Threshold Surface. This figure was created by the author to illustrate original concepts presented in this manuscript.

4 Conclusions

The informational reframing of the Navier–Stokes equations offers a viable and testable alternative formulation. By grounding speculative terms in measurable quantities and modeling their behavior under informational flow dynamics, the VTT framework becomes a bridge between classical physics and informational field theory. This perspective preserves the structural integrity of fluid dynamics while introducing novel parameters such as coherence potential and informational viscosity.

To ensure mathematical rigor and conceptual clarity, all novel terms introduced in this re-

formulation—such as informational viscosity (η_i), coherence knots (CK), and critical mass of information (CMI)—have been explicitly defined with dimensional and topological precision. The transformation of the classical Navier–Stokes equation into informational space has been conducted under the preservation of conservation symmetry, preserving its fundamental dynamics while reinterpreting physical variables through the lens of VTT.

This reformulation, which builds upon the symbolic framework initially proposed in [12], opens a promising pathway toward empirical validation.

In particular, we propose a class of controlled experiments and symbolic simulations designed to test the correspondence between informational turbulence and classical fluid instability. Such experiments include:

- Construction of synthetic coherence vector fields
- Parameter sweeps across η_i and ϕ to explore stability boundaries
- EPSV-event detection using turbulence signature recognition tools
- Interferometric and symbolic analysis of boundary-induced decoherence
- Mapping CMI thresholds to phase transitions in simulated flow fields

These experiments may be implemented using physics engines, symbolic solvers, or laboratory platforms capable of mimicking coherence-driven flow behavior. If validated, the VTT approach could offer both theoretical insight and new predictive tools for unresolved fluid dynamics problems - including turbulence onset and singularity formation in incompressible flow.

References

- [1] Fefferman, C. (2006). Existence and smoothness of the Navier–Stokes equation. In *The Millennium Prize Problems* (pp. 57–67). American Mathematical Society.
- [2] Leray, J. (1934). Sur le mouvement d'un liquide visqueux emplissant l'espace. *Acta Mathematica*, 63(1), 193–248.
- [3] Hopf, E. (1951). Über die Anfangswertaufgabe für die hydrodynamischen Grundgleichungen. *Mathematische Nachrichten*, 4(1–6), 213–231.
- [4] Ladyzhenskaya, O. A. (1963). *The Mathematical Theory of Viscous Incompressible Flow* (2nd ed.). Gordon and Breach.
- [5] Scheffer, V. (1976). Partial regularity of solutions to the Navier–Stokes equations. *Pacific Journal of Mathematics*, 66(2), 535–552.
- [6] Caffarelli, L., Kohn, R., & Nirenberg, L. (1982). Partial regularity of suitable weak solutions of the Navier–Stokes equations. *Communications on Pure and Applied Mathematics*, 35(6), 771–831.
- [7] Tao, T. (2016). Finite time blowup for an averaged three-dimensional Navier–Stokes equation. *Journal of the American Mathematical Society*, 29(3), 601–674.
- [8] Buckmaster, T., & Vicol, V. (2019). Nonuniqueness of weak solutions to the Navier–Stokes equation. *Annals of Mathematics*, 189(1), 101–144.
- [9] Strange, N. (2025). Analytic Solution of the N-Dimensional Incompressible Navier-Stokes Equations. arXiv:2501.08353. <https://arxiv.org/abs/2501.08353>
- [10] Batchelor, G. K. (2000). *An Introduction to Fluid Dynamics*. Cambridge University Press.
- [11] Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal*, 27(3), 379–423.
- [12] Bianchetti, R. (2025). The Navier-Stokes Equations Reinterpreted through Informational Geometry – A Viscous Time Theory Expansion. <https://doi.org/10.5281/zenodo.15268815>