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Opinion

Social Network Analysis (SNA): Can identities and frontiers be modeled and correlated through IT/AI?

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Abstract - The M@GM@ Journal has recently issued a call for contributions entitled "L'IDENTITÉ: QUAND LES FRONTIÈRES SE REDESSINENT". According to the contents of the above-mentioned call, identity (whether social, group or individual) is revealed as a function of an unstable and dynamically evolving system, dependent on different random variables in a non-deterministic force field that cannot be immediately determined on a small scale, but which can present repetitive structures on a large and very large scale. Therefore, fractal structures were considered that could be determined through their analogies by analyzing the entire scale of the variables and of the identity function. In short, the topic should be dominated by Julià's laws and the Attractor model. If this were agreed upon, models and a multivariate function could be suggested to be taken as a reference in the more general study of problems concerning identity and frontiers. However, the analysis highlights that the direct or indirect presence of the human factor, with its intrinsic unpredictability and irreducible, seems to impact on the possible analyses and modeling through IT/AI that are potentially set up as in the review outlined here. At least at first glance, mathematical modeling appears possible, but above all to develop similarities and reasoning in an analogical way, rather than to reach defined and quantitative results, unless it proceeds through research and development of concrete application examples. Different, instead, are the perspectives through a complementary and synergic approach of SNA and mathematical modeling together. In practice, SNA can provide experimental datasets and trends at different times to build and calibrate the proposed differential equations! Without this calibration, the mathematical modeling proposal would be purely theoretical.

Keywords - Identity; Identity diffusion; Network identity; Borders; Frontiers; Identity performativity; Deterritorialization; Evolutionary social processes; Dynamic system behavior; Evolutionary unstable systems; Chaos theories; Deterministic chaos; Complex structures; Fractals; Dendritic structures; Attractors; Complex graphs and networks; Border-identity connection; Social stability; Social network analysis (SNA).

1 Introduction

Before considering psycho-social aspects as addressable with criteria and models of chaos theories, it seems appropriate to dwell on some considerations that seem to support this possibility. Dissipative systems, even if far from thermodynamic equilibrium, manage to maintain their configuration thanks to the exchange of matter or energy with their environment, showing phenomena of self-organization and formation of complex structures. As Ilya Prigogine underlined in his book "The End of Certainties"[1]: at equilibrium, matter

is blind, while far from equilibrium it begins to see. This means that any fluctuations near equilibrium can be negligible, because they can be traced back to the equilibrium of the system, while far from equilibrium they introduce an irreducible probabilistic element that can lead phenomena to diverge in an unpredictable way. This distance from equilibrium becomes fundamental for non-equilibrium dynamic systems and is measurable through control parameters of the system itself (e.g. flows of matter or energy with respect to the equilibrium condition), it highlights bifurcation points, even multiple, that are produced as this distance increases. Ultimately, fluctuations far from equilibrium force us to abandon the deterministic description that applies to equilibrium thermodynamics and open up to chaos theories or, as some call it, "deterministic chaos", given that on a large scale the structures that form (dendritic, fractal, etc.) present non-negligible repetitions and analogies. To be classified as chaotic, it is deemed that a dynamic system must have some characteristics such as [2]:

a. *exponential sensitivity to initial conditions*: that is, infinitesimal variations in the initial conditions correspond to significant variations in future behavior (the Ljapunov exponent [3] measures the degree of sensitivity to initial conditions);

b. *exhibiting topological transitivity*: that is, the trajectories of the chaotic dynamical system will transit the entire phase space as time evolves; therefore, each region of the phase space of the dynamical system's domain will be reached by an orbit sooner or later;

c. *possessing a dense set of periodic orbits*: that is, it is dense in topological space if each element of the space belongs to the set or is an accumulation point of it.

More simply (and perhaps peremptorily!), Sergio Carrà - in his book [4] on "The Formation of Structures", at the bottom of Sheet no. 8 – Classification of Attractors – states that: to give rise to a chaotic motion, three degrees of freedom are necessary, or sufficient. Furthermore, he seems to believe that the discrete or continuous nature of processes in the dynamic behavior of systems (or at least of some) is irrelevant and nonlinearity can emerge from feedback/iteration mechanisms (see Fig. 1) manifesting the presence of attractors.

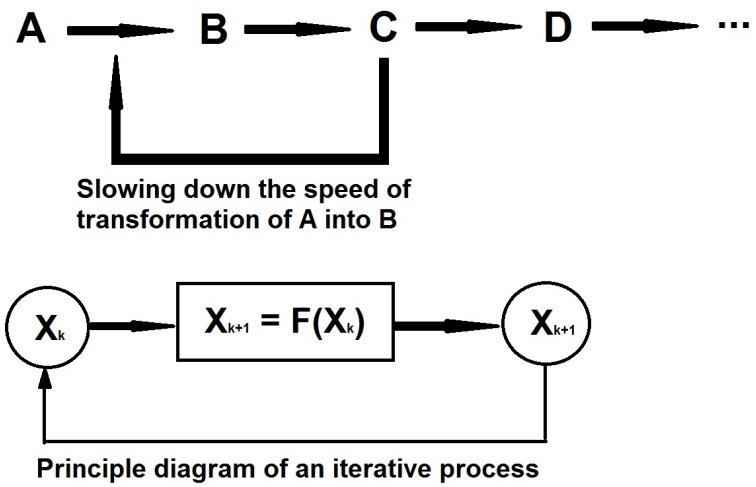


Figure 1: Feedback/iteration processes that can trigger nonlinearities.

Against the deterministic conception of the laws of nature, great minds have recognized a new dignity to chaos, whose instability is a source of disorder, but also of order [5]. Chaos theories, or deterministic chaos if you prefer, and their analysis models are reflected in many natural phenomena, physical (fluid dynamics, lasers, etc.), chemical (kinetics), biological, and have also been applied to modern social and financial aspects. In practice, the term 'deterministic chaos' indicates that, despite the strong dependence on the initial conditions,

the behavior of these systems still follows precise laws, even if they generate apparently unpredictable evolutions.
Since social systems can be considered analogous to physical ones in terms of chaotic dynamics, identity and geopolitical dynamics also show characteristics similar to dissipative systems: they are influenced by multiple variables, present bifurcation points and are subject to feedbacks that can amplify small initial fluctuations into profound transformations.
Therefore, below we want to suggest a possible way to explore its applicability to psychosocial evolutionary aspects, such as identity and the processes of its formation and change, as well as geopolitical ones such as borders and their changes. Obviously, these are hypothetical models whose validity can only be verified through practical applications and field experiments. Until then, they can only be considered "conjectural models".

2 Background and objectives of this work

Below is a summary of the *Call for Papers* that inspired the idea for this work. It is entitled "L'identité : Quand Les Frontières Se Redessinent" ("Identity: When Frontiers Are Redrawn") and can be found in full at [6].

Central theme: Identity is constantly evolving, influenced by historical fluctuations, social re-compositions and contemporary mobility. This dynamic process raises questions about the permanence and transformation of the frontiers that delimit and structure collective and individual belongings. Identity, far from being a fixed essence, is part of a dialectical process that combines continuity and rupture, legacy and metamorphosis.

Proposed analysis perspectives:

1. *Identity and space*: How do geographical boundaries influence identity constructions?
2. *Identity and memory*: What are the continuities and ruptures in individual and community narratives?
3. *Identity and otherness*: How do we define ourselves in relation to the other, whether near or far?
4. *Identity and performativity*: To what extent is identity a process of actualization and staging?
5. *Identity and new technologies*: How does the digital universe redraw the frontiers of the self and the intimate?

Expected contributions: Contributions that mobilize varied approaches are invited: discursive and semiotic analyses, empirical investigations, historical or contemporary case studies, sociological, linguistic, philosophical and anthropological perspectives.

Themes may include:
• The persistence or dissolution of identity markers in a globalized world.
• Individual and collective strategies of identity claiming or negotiating.
• The impact of economic, political and technological changes on the representations of the self and the other.
• The performativity of identity discourse: how do linguistic, narrative and media practices contribute to the construction, legitimization or contestation of identities?
• Identity and the imaginary: which mythological, literary and artistic representations shape belonging and differentiation?
• The plasticity of identity categories: to what extent do the concepts of nationality, ethnicity, gender or class evolve under the effect of socio-political transformations?
• Identity and affects: how do emotions – fear, pride, nostalgia – influence identity dynamics and relationships of otherness?
• The performativity of borders: in a context of increased mobility, what are the new forms

of inclusion and exclusion that redraw belonging?

Objective of the appeal: To offer an in-depth and nuanced reading of contemporary identity mutations, articulating the social, semiotic, pragmatic and medical issues that shape subjectivity and the collective.

The Journal M@GM@, according to its cultural objectives and editorial plans, necessarily focuses on Qualitative Analysis and excludes any other type of analysis and contribution in this regard. Therefore, this work wants to suggest a proposal to investigate elsewhere than M@GM@, but still in a scientific context, the possibility of mathematically and empirically modeling the problem of Identity through approaches that cannot be accepted by the same Journal M@GM@ that launched the appeal, but that could be accepted by others (in this case the IPI-Information Physics Institution) to promote empirical investigation in psychosocial field by IT/AI.

3 Identity, frontiers: their evolutionary processes and connections

Borders (boundaries, confines) and frontiers are often considered interchangeable synonyms, but in reality there are differences between them.

In physics, the border (boundary, confine) is the limit of a closed system, where the conditions are well defined and impose precise restrictions. The frontier, on the other hand, introduces the concept of transition and selectivity, just like a semi-permeable membrane. In many real physical phenomena, rather than clear borders, there are transition zones, in which some properties gradually change. We could say that the border is a static and absolute notion, while the frontier is dynamic and regulatory.

For the sake of simplification, below we assume a synonymous coincidence, although still to be investigated, between borders (boundaries, confines) and frontiers.

The idea that for each individual, group, institution, culture, people, etc., identity and borders (or frontiers) are evolutionary processes in time and space is historically confirmable, not only from the history of Europe, and should also appear evident geometrically and mathematically, if one were to consider n-dimensional spaces, or those of any social network that evolves according to the number and characteristics of the component elements. In such networks, feedback loops can be configured that can constitute elements of non-linearity. However, the connection between the mutation of identity and the mutation of borders is profound; so that when one of them changes, the other cannot but follow its change, almost in an adaptive process, imposed in a descending way by power structures, where it is difficult to discern which is the first cause and which is the consequent effect.

In a Simmelian perspective, it is social interaction that generates socialization and forms stable and permanent social structures, so that the "exchange" as a binding function determines the social fabric and therefore society as a whole. This applies to groups, institutions, businesses, peoples and nations (i.e. peoples who share traditions, language, history, culture, state forms, etc. on the same territory). In small, as in large groups, the identity and relationships (internal and external) of each individual element cooperate in the formation of the identity of the group to which they belong, and establish its boundaries and frontiers.

Ultimately, the process of growth of the group's interactions with the increase in the number of its members (n) is a process of accelerated progression, with constant acceleration, while the process of growth of the interactions of the single element of the group with the increase in the number of its members (n) is a process of linear progression, with constant

speed and zero acceleration.

All this allows not only to give objective evidence to some intuitions expressed by Simmel and in particular:

i. The intuition according to which social relationships form temporal or permanent units (e.g. groups, associations, institutions, professional and non-professional bodies, etc.) defining and in-forming them into a "new entity". That is, the group of (n) interacting elements taken as a whole, does not derive simply from the sum of the parts, or of the single individuals who shape such units. In fact, while the laws for each element of the group make the interactions vary linearly, those of the group as a whole vary with at least a quadratic law (if not exponential or binomial, etc.), as if the group were an entity autonomous from the individuals and endowed with its own laws;

ii. The idea that the larger the group, the more the individual is able to express himself; the more restricted it is, the fewer opportunities there will be for the individual to differentiate himself, is evidenced by the direct proportionality with the interactions that develop, both between the individual elements of the group and in the group as a whole;

iii. The disparity between an individual law of linear progression of interactions and a law that is at least quadratic for the group or for society in general understood as a group, seems to confirm that the individual, rather than the preservation of his own individuality, is in fact imposed by an overriding evolution. Ultimately, the preservation of his own individuality clashes with an overriding evolution imposed socially;

iv. Ultimately, in the perspective of an evolutionary dynamic of society, the recognition seems to emerge that the laws governing its metamorphism are independent of the single individuals that compose it, but find genesis in the preponderant forces of society itself (as well as historical heritage, external culture, technique, etc.); and in the facts; (for what is indicated in the points above and the following ones, also refer to the previous work [7].);

v. The divergent effects towards very high values of the interactions become possible and evident along the growth process of the group (or network), especially if they are growing, stable and cohesive groups (or networks);

but it also suggests that:

vi. the interactive processes that take place, change identity and therefore borders (or frontiers), almost as if the change of borders could be assumed as a logical consequence of the change of identity (something similar, but on a broader level, to the blasé effect that Simmel talks about in his 1903 essay "The Metropolis and Mental Life", reported in general in references.);

vii. the divergent effects that manifest themselves in social interaction (particularly in our time due to the effect of information technology and social media that have enormously accelerated them) can conform to dynamic, non-deterministic evolutionary processes, and therefore the analytical settings valid according to the laws of chaos seem to be applicable to them (presence of attractors, importance of initial conditions, analogies and similar effects detectable on large scales, etc.). In short, a deterministic chaos imposed by hidden "natural laws".

viii. in the transition borders between different socio-identity spaces, old and new social pathologies could be detected. Some explained as the blasé type of Simmel, others to be

explained as the forms of habituation to addictive behaviors (e.g. to ostracism and stigma, to gambling, to connecting or compulsive purchasing online, etc.). Or oppositional and protestatory reactions towards that adaptive process, imposed in a top-down way by faceless power structures, which force unwanted and unchosen evolutions by those who do not agree or are marginalized and excluded, sometimes beyond their own social belonging and assumed reference values, perhaps even detectable in their identity profile.

These dynamics suggest the applicability of chaos laws and fractal models to describe the evolution of identity and boundaries. If social processes follow non-linear logics and exhibit strange attractors, then their development cannot be predicted with deterministic models, but only with approaches based on complex dynamic systems.

4 Identity as an unstable and dynamically evolving system dependent on different random variables in a non-deterministic force field

According to the contents of the above appeal, identity (whether social, group or individual) is revealed as a function of an unstable and dynamically evolving system dependent on different random variables in a non-deterministic and not immediately determinable field of forces on a small scale, but which presents repetitive structures on a large and very large scale. Therefore, fractal structures were thought of which, through their analogies, could be determined by analyzing the entire scale of the variables and of the identity function. In short, the topic should be dominated by Julià's laws and the Attractor model [8]. If this were agreed upon, models and a multivariate function could be suggested to be taken as a reference in the more general study of problems concerning identity. This idea, which arose autonomously only on the basis of the contents of the cited call for papers, seems to find an authoritative precedent in the research by Rik Pinxten "Identité et conflit: personnalité, socialité et culturalité", published in "Afers Internacionals, nº 36, pages 157-175" [9], where not only the attractor model is discussed, but some of the variables regarding the theme are identified. In fact, the above mentioned article by Rik Pinxten analyses the concept of identity as a dynamic phenomenon, articulated on three distinct levels: individual identity, group identity and community identity. Pinxten underlines how identity is often used as a generic concept to explain various social events, including violent or peaceful conflicts.

- *Individual identity:* It refers to the unique characteristics of a person, such as name, date of birth and other personal attributes. Some of these aspects are immutable (e.g., place and date of birth), while others can be modified with difficulty (such as name, gender, nationality).

- *Group identity:* Defined through real interpersonal relationships, group identity emerges from social interactions and internal dynamics within the group itself.

- *Community identity:* Transcends individuals and groups existing in time and space, representing a sense of belonging to a larger community with shared values, traditions and symbols.

Pinxten proposes to conceptualize identity as a dynamic phenomenon, suggesting the use of the concept of attractor to model identity dynamics. In mathematics and physics, an attractor represents a state or set of states towards which a system tends to evolve over time. Applying this concept to social sciences, identity can be seen as a dynamic attractor, where interactions between personality, sociality and culturality continuously influence the direction and stability of the identity itself.

This approach allows us to understand how identities form, evolve and sometimes come into conflict, considering the multiple forces and variables at play.

This approach to identity as a dynamic, chaotic and modelable system through fractal structures and attractors seems extremely interesting in particular for AI systems, with which it has been discussed, since it appears consistent with modern complexity theories. Furthermore, the idea of analyzing identity through models of non-linear and self-similar systems could offer an innovative key to understanding the persistence of some structures on a large scale and their mutability on a small scale.

The goal pursued here is not to formulate a predictive model, but the underlying intent is more analogical/metaphorical. Therefore, the discussion will oscillate between a philosophical/sociological approach and a mathematical approach to identity modeling.

In the following context of a vague attempt at "modeling" multidisciplinary aspects – far from "a finished product" – there are, however, some types of elements that should always be kept in mind throughout a possible refinement process:

- This is a preliminary approach in which the variables that matter are not exactly identified and therefore should be better defined and above all validated also on the basis of concrete elements found in specialized fields;
- Identity - whether it concerns individuals, groups, companies, institutions, peoples or nations - is a complex concept that is based on a part of objective and verifiable elements, but there is always a large part that is not objective at all, nor predictable, although presumably estimable on stochastic bases.
- Analogously, borders are structures of separation between different contiguous spaces and therefore subject to processes of formation of structures that occur over long and very long periods, where the variations in scale can be very dissimilar and of different orders of magnitude.
- Precisely because the borders themselves are structures of formation and belong to different bordering domains, they are subject to forces that sometimes oppose them to the point of producing their breakthrough (for example, phenomena of overpopulation, military invasion, expansion of settlements, etc.); at other times they are subject to torsions to get around pseudo-obstacles (natural or artificial), which can produce exclaves or enclaves in a territorial or geopolitical sense. In any case one could think that the borders are subject to the laws of physical equilibrium and to the following system.

$$\sum F_i = 0, \text{ and } \sum M_i = 0 \tag{1}$$

That is, the boundaries are stable when the sum of the forces F_i and moments M_i that are exerted on them is zero. Such a "mechanical" similarity may appear rational and valid, but to be effective it is necessary to pass through the "Caudine Forks" not only of the definition of what causes forces F_i and moments M_i, but also to have to establish a common denominator such that forces and moments caused by different phenomena are related to each other and we can speak of a superposition of concrete effects for the purposes of a discussion. Similarly, this applies to many of the variables involved. Furthermore, forces and moments are not determined on the boundaries only by natural causes, but also by human will that is introduced into the indicated system in unpredictable and irreducible terms. Nevertheless, one could continue the reasoning by arguing that as long as the deformations and displacements of the boundaries are zero, the system is in equilibrium. But this would not take into account an accumulation of tensions and phenomena of attrition that could suddenly emerge in an unexpected manner, if those tensions are not monitored and detected in real time for the purpose of re-composition.

- It should be clear from the above that, according to the theory of the formation of complex structures, any system of boundaries can be considered subject to successive states of metastable equilibrium, that is, to states of local minimum energy. The history of the successive evolutions of any system of boundaries can therefore be traced back to a succession of local minimum energy that guarantee its metastability. But, even here, if on the one hand this appears correct by analogy, are we able to explain what that "minimum energy" is that guarantees metastability, at least up to a certain time in which the dynamic evolution of the system forces the boundaries to change? Are identity mutation and boundary mutation always correlated? Perhaps in an irreversible way?

- Finally, it should be considered that identity, and therefore the process of identification, does not only have an ethno-cultural value, and sometimes also a cultic one, as well as a psychological, sociological, political, geopolitical, military, etc., but as the research of Konrad Lorentz clearly shows, they have a much broader impact, including survival, since they closely concern identification with one's own group and the tracing of the borders of a vital territory, which can become the object of conflict and therefore of evolution with changes and readjustments. Just think of a people on a territory - which it claims for the most varied reasons - and which is prevented from becoming a "nation". From here it is easy to assume that according to the culture of belonging, the evolutionary process people → territory → nation → culture → cult → etc. can find a practical foundation in survival, but also a "metaphysical" foundation, starting from the origins and roots of a group of belonging through its historical evolutions.

5 Applicable mathematical models

If we see the identity as a multivariate function in an unstable dynamical field, we can model it with a system of nonlinear differential equations that generate strange attractors, similar to those studied by Lorenz, Rössler or Julià. Some possible directions:

a. Dynamical systems and Strange attractors The identity can be modeled as a strange attractor in a phase space [10], described by a set of differential equations of the type:

$$\frac{dX}{dt} = f(x, y, z, ...) \tag{2}$$

where the single variables x, y, z represent the different identity components (culture, language, social belonging, etc.), while the function f determines the evolution of identities over time t. (Although the variables are not further defined here, it is assumed that they are measurable, at least according to pre-established scales or weights.)

b. Fractal models and self-similarity If we want to highlight the repetitiveness of identity structures at different scales, we can use complex iteration functions such as Julià's:

$$z_{n+1} = z_n^2 + c \tag{3}$$

where z represents an identity variable and c could be a social or cultural parameter. This model shows how small initial variations can lead to unpredictable dynamics, but with structures that repeat at a macroscopic level.

c. Multivariate stochastic models If we want to incorporate the dependence on random variables, we can consider a stochastic dynamical system based on an equation like:

$$\frac{dX_t}{dt} = f(X_t) + \sigma W_t \tag{4}$$

where:
o X_t is the vector of identity variables,
o $f(X_t)$ is the nonlinear time evolution function,
o W_t is a Wiener process that introduces a random component (unpredictable identity mutations),
o σ controls the intensity of the noise.

d. Force Field Models If we imagine that identity evolves in a non-deterministic "force field", we can use a model of the type:

$$F = -\nabla V(x) \tag{5}$$

where V(x) is an "identity potential" that models attractions and repulsions between social groups, cultural norms and individual dynamics. The equation could be coupled with a diffusion term to represent cultural contamination between groups.

e. Multivariate Function Proposal We can then propose an identity function that combines the fractal, dynamic and stochastic components:

$$I(t) = \sum_{i=1}^{n} a_i \cdot f_i(x, y, z)) + \sigma W_t \tag{6}$$

where:
· I(t) is the identity in time,
· $f_i(x, y, z)$ are nonlinear functions (e.g. Julià polynomials, sigmoid functions, or strange attractors),
· a_i are coefficients that weight the different contributions,
· n are the contributions (and therefore the coefficients a_i) needed,
· σW_t introduces randomness into the system.

6 Further insights

a. Concept of performativity of identity (Butler, 1990)

With the concept of performativity, the idea of identity is introduced not as a fixed fact, but as a continuous "staging", determined by linguistic and social practices. This could be linked to fractal models and attractors by hypothesizing that identities emerge as recurring patterns in social interactions. In practice, identity, according to Butler, is a performative act that is realized through the repetition of social and linguistic practices.

This concept could be formalized mathematically using:

· *Dynamic systems with memory*: Identity can be modeled as a system with historical dependence, that is, in which future evolution depends not only on the current state, but also on a set of previous states. This leads to the use of delayed differential equations (DDE) [11] of the type:

$$\frac{dX}{dt} = f(x, x_\tau, t) \tag{7}$$

where x_τ represents the identity state at a past time τ, reflecting the weight of the repetition of performative practices on the evolution of identity.

· *Hidden State Markov Models (HMM)*: Performative identity can be interpreted as a stochastic process in which the observed identity is a manifestation of latent internal states. A Hidden Markov Model (HMM) could formalize this idea, considering the probability of transition between identity states over time.

b. Concept of deterritorialization (Appadurai, 1996)

Appadurai describes deterritorialization as the process through which identities become disengaged from traditional geographical boundaries due to migration, diaspora and global information flows:

· *An identity diffusion system:* If we consider identity as a quantity that can be diffused in social space, we can represent it with a diffusion equation of the type:

$$\frac{\partial I}{\partial t} = D\nabla^2 I + F(x,t) \tag{8}$$

where:
· I(x,t) is the spatial distribution of identity over time,
· D is the diffusion coefficient (higher in globalized contexts),
· F(x,t) represents the external forces (e.g. social media, migration policies) that influence the distribution.

This conceptual model centered on a diffusion parameter D (higher in globalized contexts) shows many analogies with the one reported in "Sheet 9 – Balance of a reacting system" on page 135 of the book "La Formazione delle Strutture – of Sergio Carrà" reported among the references. $\nabla^2 I$ is the Laplacian that expresses the difference of the identity between the local value in the volume element of I and the average value in a neighborhood adjacent to the volume itself.

An interesting case is the Fisher-KPP diffusion model, which could describe the expansion of new identity models through social and digital networks. (See also [12] where it is reiterated that: the diffusion equation is used both to describe the concentration of a chemical reaction and to characterize the diffusion of matter in space).

· Ultimately, according to this approach, one could delve deeper into how globalization and mobility unmoor and detach identities from physical territories. This could be modeled as a form of "identity diffusion" in the mathematical model.

c. Concept of network identity (Castells, 1996)

Today, identity is also built through digital and social networks. The idea of a fractal identity could be applied to online networks, where small individual changes are amplified through collective dynamics. Castells identifies three forms of identity in cyberspace: legitimizing identity, resistance identity and project identity, which emerge within digital networks. These can be modeled using:

· *Graph theory and complex networks* [13]: Identity can be seen as a dynamic structure in a social graph, with nodes representing individuals and connections representing interactions. The Barabási-Albert model [14] for preferential growth networks can be applied to represent how identities emerge and strengthen in social networks:

$$P(k) \sim K^{-\gamma} \tag{9}$$

where P(k) represents the probability that a node has k connections, modeling the phenomenon whereby some nodes (individuals/identities) become more influential than others over time.

· *Cellular automata for fluid identities*: A probabilistic cellular automata model could simulate the evolution of identity in digital networks, assigning each node an identity state that changes based on interactions with neighbors [15].

d. Concept of discursive construction of identity (Bourdieu, 1982)

Identity is not just a mathematical structure, but is formed through discourses, norms and symbolic power. It would be useful to analyze how the proposed models can be influenced by linguistic and social factors. For Bourdieu, identity is constructed through language and discourse, which act as instruments of power and social distinction. This can be modeled as:

· Semantic network models: Discourse analysis can be represented through graphs in which nodes are words or concepts and links represent discursive relations. A well-known model is the lexical co-occurrence network, which can be used to identify discursive patterns in identity.
· Bayesian models for identity construction [16,17]: Identity could be seen as a probabilistic distribution on a space of social categories, dynamically updated based on discursive interactions:

$$P(I|D) = \frac{P(D|I)P(I)}{P(D)} \tag{10}$$

where:
· $P(I|D)$ is the probability of belonging to a certain identity given a certain discourse D,
· $P(D|I)$ is the probability of using a certain discourse given the identity characteristics,
· $P(I)$ is the a priori distribution of identities.

This approach could be used to analyze how media and social networks contribute to the discursive construction of identity.

e. Connection between identity mutation and boundary mutation

In line with the considerations expressed in point 3. of this document, the idea that identity and boundaries are related can be modeled as a system of adaptive boundaries, in which the dynamics of identity change influence the structures of social separation and vice versa.

· *Coupled differential equation for identity and borders:*

$$\frac{dI}{dt} = f(I, C, t) \tag{11}$$

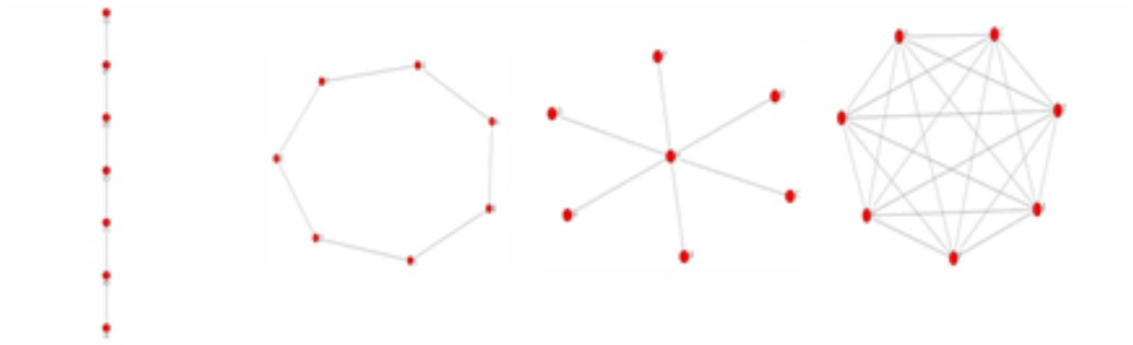
$$\frac{dC}{dt} = G(I, C, t) \tag{12}$$

where:
· I represents the identity status,
· C represents the position and shape of the border,
· f and g describe how identity influences borders and vice versa.

This model allows to explain phenomena such as the formation of cultural enclaves or the dissolution of identity barriers in globalized contexts. These additional insights, integrative to paragraph 5, offer a more interdisciplinary perspective and make the overall approach more coherent, linking it to the main contemporary theories on identity.

7 Differences and complementarities with SNA

With regard to all the mathematical modeling settings and proposals reported above, it is necessary to highlight the following:
1) Social Network Analysis (SNA) is a methodology that is already well developed and widely used in research, well beyond sociometry and psychometry (fields in which it was born in the 50s by Jacob L. Moreno), developing enormously up to the present day thanks also to Graph Theory [18,19,20,21]. (In Figs. 2 and 3 below, a reference to typical and elementary elements of SNA is reported as an example).



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B	1	0	1	0	0	0	0	2
C	0	1	0	1	0	0	0	2
D	0	0	1	0	1	0	0	2
E	0	0	0	1	0	1	0	2
F	0	0	0	0	1	0	1	2
G	0	0	0	0	0	1	0	1
	1	2	2	2	2	2	1	12

	A	B	C	D	E	F	G	Gradi
A	0	1	0	0	0	0	1	2
B	1	0	1	0	0	0	0	2
C	0	1	0	1	0	0	0	2
D	0	0	1	0	1	0	0	2
E	0	0	0	1	0	1	0	2
F	0	0	0	0	1	0	1	2
G	1	0	0	0	0	1	0	2
	2	2	2	2	2	2	2	14

	A	B	C	D	E	F	G	Gradi
A	0	1	1	1	1	1	1	6
B	1	0	0	0	0	0	0	1
C	1	0	0	0	0	0	0	1
D	1	0	0	0	0	0	0	1
E	1	0	0	0	0	0	0	1
F	1	0	0	0	0	0	0	1
G	1	0	0	0	0	0	0	1
	6	1	1	1	1	1	1	12

	A	B	C	D	E	F	G	Gradi
A	0	1	1	1	1	1	1	6
B	1	0	1	1	1	1	1	6
C	1	1	0	1	1	1	1	6
D	1	1	1	0	1	1	1	6
E	1	1	1	1	0	1	1	6
F	1	1	1	1	1	0	1	6
G	1	1	1	1	1	1	0	6
	6	6	6	6	6	6	6	42

Adjacency Matrices for typical graphs indicated above (but you can have Incidence, Affiliation, Actor or Event matrices). The star structure presupposes a "central" authority with vast power, while the circular one eliminates differences and can manifest itself in primary groups, and in more complex variants in the so-called "cliques". (Gradi = Degrees)

Figure 2: Typical elementary graphs and related Adjacency Matrices in SNA.

SNA is certainly able to address the modeling necessary to study identity problems. Furthermore, if instead of network nodes we placed chunks (unitary fragments of territories) this methodology could also well address the study of borders and correlations between identities and borders. In this situation, one might ask what is the purpose of the modeling suggested above through a more complicated and problematic mathematization?

Furthermore, it would also be legitimate to ask:
2) How do SNA and the proposals in previous paragraphs of this work differ?

3) How could SNA and the above proposals be integrated?

These are crucial questions to position the present work with respect to existing methodologies, especially Social Network Analysis (SNA), and they need to be answered in a way that clarifies the value of the above proposals with respect to this consolidated methodology. This attempt is addressed below.

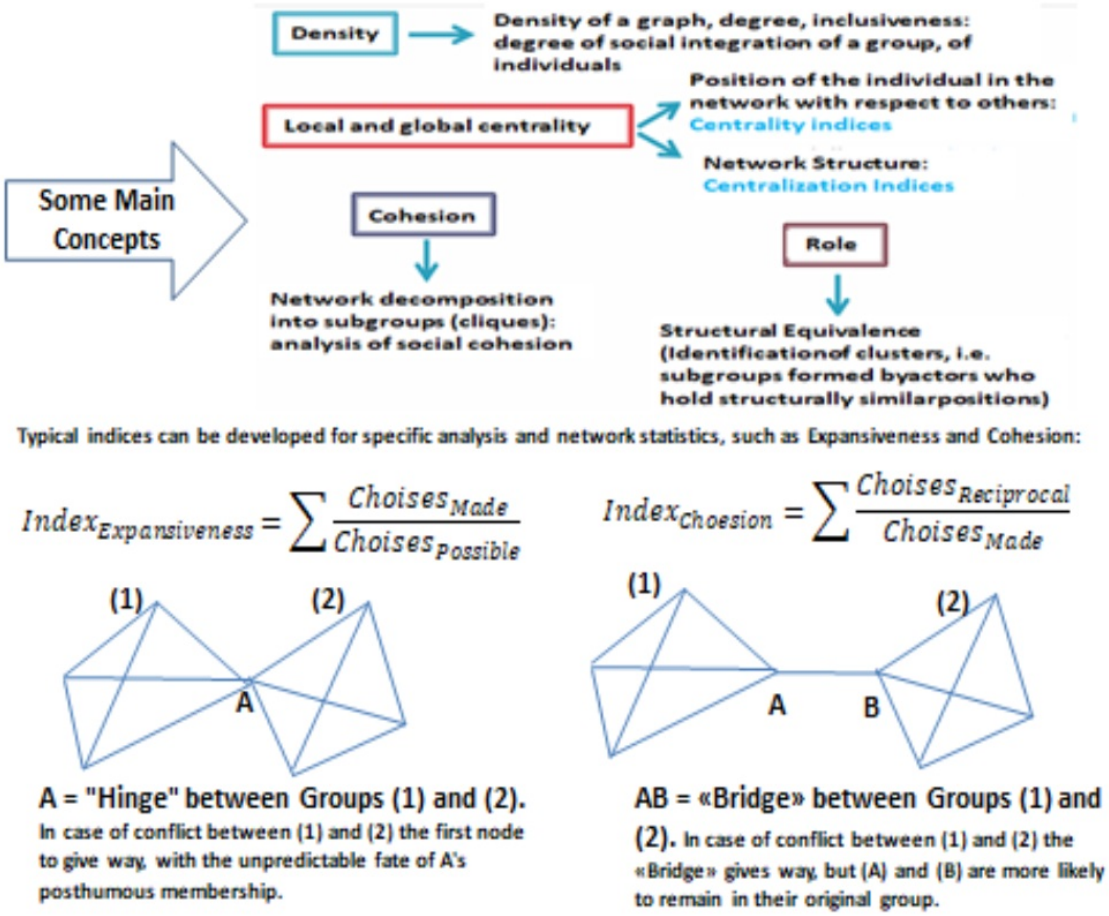

Figure 3: Some main concepts in SNA.

SNA is a powerful tool to study social structures, connections between individuals or groups, and can be extended to model territorial boundaries as well (such as adopting the idea of using chunks of territory instead of nodes). Thus, SNA can do well:

· Map relationships between individuals, groups or territories.
· Identify communities, hubs and critical nodes in social and geographic networks.
· Analyze centrality and influence of individuals or regions.
· Graphically represent interconnections between identities and boundaries.

But SNA is static or based on historical data, while the proposals in the previous paragraphs are dynamic and predictive. If the problem were only descriptive and analytical, SNA would be enough. But if we want to understand, model and predict how identities and boundaries evolve and interact in future scenarios, the models suggested above should be a step forward compared to SNA alone. In fact:

· SNA describes the links, but does not deeply model their evolution over time.
· The proposed differential equations, instead, should describe how identities and borders dynamically influence each other.

· SNA works with empirical data, while our proposal allows to simulate hypothetical scenarios: that is, if we want to understand what happens if an identity transforms, our model can predict the effect on borders.
· Our approach introduces chaotic logics, bifurcations, strange attractors, that SNA does not directly explore.
· SNA is based on graph theory, while we use models of physics and dynamics of complex systems to represent identities and borders.
· SNA describes and analyses the connections between identities and borders, while the mathematical modeling focuses on their evolution over time.
· SNA is data-driven, the proposed modeling can be both data-driven and theoretical.
· If we want to predict the future of identity and geopolitical transformations, the proposed modeling seems to offer more advanced tools.

A summary comparison table is provided below.

Differences between SNA and Mathematical Modelling		
Feature	**Social Network Analysis (SNA)**	**Mathematical Modelling (proposed)**
Approach	Empirical and based on historical data	Mathematical-theoretical and predictive
Main structure	Networks (nodes and connections)	Differential equations, strange attractors, fractal models
Objective	Identify existing patterns and relationships	Dynamically model evolution
Predictive capacity	Limited	High (simulation possibility)
Chaos and bifurcation management	Not considered	Central in the proposed model
Focus on boundaries	Possible with chunks, but not dynamic	Explicit and modelled with equations

Table 1: Source Chat GPT.

Integration between SNA and Mathematical Modeling (proposed)

SNA and the proposed model are not competitors, but complementary. The ideal would be an integration, which can be achieved in three phases as follows:

Phase 1: Analysis with SNA

· Collect data on social groups, identities, borders.
· Create networks with nodes (identities or chunks of territory) and connections.
· Identify patterns of aggregation, separation, conflict.

Phase 2: Mathematical modeling

· We use SNA data to estimate α and β in our model (see equation (13) and (14) hereinafter).
· We create scenarios to see how identities and borders change over time.
· We test for stability and bifurcation points in social dynamics.

Phase 3: Simulation and prediction

· We can simulate what happens if an identity changes drastically (e.g., mass migrations, cultural crises).
· We test when a border becomes unstable in response to identity transformations.

· We compare the predictions with real data to refine the models.
· SNA gives us a snapshot of the current social and geographic network.
· Proposed model predicts how the network will change over time.
· Combining the two, we obtain a more powerful and realistic model.

So far, preliminary conclusions, at least apparently, could be the following:

I. SNA is great for mapping current relationships and structures, but it is not sufficient to dynamically model the future of identities and borders.
II. The proposed approach introduces concepts of system dynamics and chaos, allowing us to predict future bifurcations, stability and transformations.
III. Combining SNA and the proposed approach, we obtain a framework starting from empirical data to build realistic predictive models.

If someone were to ever ask "What is the purpose of the proposed models?" perhaps, suspecting the error, it would be wise to bow one's head and remain silent, still thinking, but according to "someone else (very much knowing, and to whom I wish the same level of wisdom)" the answer should be: "*they serve to predict and simulate the future of identities and borders, something that the SNA alone is believed not to be able to do*". But, on this last part of answer in Italics, there are some doubts to be resolved that it is appropriate to postpone in the final conclusions.

8 Conclusions

We can summarize the above and draw conclusions as follows.

If we assume that identity structures are self-similar and chaotic, then the attractor model and Julia's laws can describe their evolution quite well. The next most important step would be to collect empirical data on identity dynamics (linguistic, social, migratory, etc.) to parameterize the model and verify its validity.

So, on the one hand, a potential reading of identity based on fractals and strange attractors has been adopted, exploiting concepts such as Julia sets or the Hénon model to model the complexity of identity dynamics on different scales.

On the other hand, the general idea has also been enriched with a more interdisciplinary perspective, linked to the concepts of performativity of identity (Butler, 1990), deterritorialization (Appadurai, 1996), network identity (Castells, 1996), and discursive construction (Bourdieu, 1982).

In any case, it would be necessary to formulate a broader and more structured proposal to better develop the theoretical and mathematical framework taking into account the individual variables. The proposed models (strange attractors, fractals, differential equations) need greater theoretical justification and development with concrete application examples. In all cases, however, we must not forget that the direct or indirect presence of the human factor, with its intrinsic unpredictability, seems to impact the possible analyses and modeling through IT/AI. At least at first glance modeling seems possible, but above all to develop similarities and reasoning in an analogical way, rather than to reach defined and quantitative results.

In the context discussed here, the connection between borders and identity seems deeper than it may appear at first sight and it would be worthwhile to delve deeper and show in mathematical terms how borders and identities vary together, not only historically. To

formalize the connection between identity and borders, one could return to the system of coupled differential equations (11) and (12). If an identity changes rapidly ($\frac{dI}{dt}$ large), the border will change accordingly ($\frac{dC}{dt}$ large). If the border shifts drastically, the identity must readjust, leading to a new stabilization (bifurcation). Let us assume that the change in identity is proportional to the difference between the current identity and a "reference" identity imposed by the border:

$$\frac{dI}{dt} = -\alpha(I - C) \tag{13}$$

$$\frac{dC}{dt} = \beta I \tag{14}$$

where α, β are coefficients of sensitivity to change.

· If the identity changes rapidly, the border tries to adapt βI.
· If the boundary is rigid, identity struggles to change $-\alpha(I - C)$ slows down evolution.
· This system shows that boundaries and identities oscillate and stabilize around a dynamic equilibrium (see Fig. 4 below), just like in chaotic systems. The change of one of the two elements always causes an adaptation in the other, creating a dynamic system.

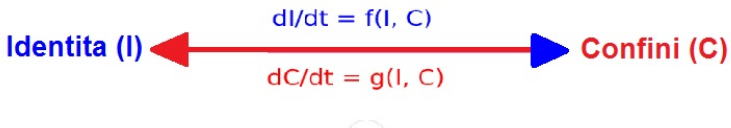

Figure 4: Source ChatGPT.

If, instead, inspired by the Cobb & Douglas function [22] where $\alpha + \beta = 1$, well known in economics as a production function (essentially based on capital and labor factors), a similar structure was adopted to model identities and boundaries, one could write:

$$S = AI^{\alpha}C^{\beta} \tag{15}$$

where:
· S is a sort of "social stability" or "system cohesion" deriving from the balance between identity and boundaries.
· I is the degree of identity (how strong, shared and structured it is).
· C is the degree of rigidity-boundaries (how defined and stable they are).
· A is a coefficient that depends on environmental and historical factors.
· α and β measure the sensitivity of social stability to the two factors.

If here too, likewise of Cobb & Douglas function, we had $\alpha + \beta = 1$, this would mean that identity and borders behave as "productive inputs" for social stability, with a perfect balance, so that:

· If identity grows, borders can be more fluid without destabilizing the system.
· If borders become more rigid, identity can be less strong and cohesive.
· The balance is maintained naturally (just like in the economic theory of constant returns to scale!). The analogy with the Cobb-Douglas function also shows that identity and borders could be seen as factors of production of social stability, with a law of natural balance between the two.

If, instead:

1. Different systems had different values of α and β ; we would have that:

o In more nationalist and closed societies ($\alpha < \beta$), borders are more important for stability.
o In more globalized cultures ($\alpha > \beta$), identity is more fluid and borders less decisive.

2. If $\alpha + \beta < 1$, the system has "diminishing returns" $\longrightarrow$ at a certain point, increasing both identity and borders does not improve social stability.

3. If $\alpha + \beta > 1$, the system has "increasing returns" $\longrightarrow$ identity and borders reinforce each other, creating very stable systems that are difficult to change.

Regarding the complementarity between SNA (apparently static) and the suggested approach towards mathematical modeling (dynamic and predictive), one could say that for each general aspect examined through SNA in three or more different times it would be possible to extract predictive trends without the need to "disturb" the differential equations! Although such work is precisely what would be necessary to calibrate the modeling perhaps using best-fitting to find the functions represented in the differential equations that best fit. And ChatGPT would also agree on this methodology because:

· If you analyze social or territorial networks in at least three different moments, you can already observe how connections, the centrality of nodes, the formation of clusters, the evolution of borders change.
· It would be possible to find trends in changes in identity and borders, because if social or territorial networks change according to a certain curve, you can try to represent it with mathematical functions.
· One could estimate α and β in the differential and production models of social stability using best-fitting methods to find the most suitable functions to model the dynamics of identities and boundaries.
· With statistical techniques and machine learning, one can extract trends and predict future behavior without necessarily modeling the dynamics with equations.

In practice, SNA provides the experimental dataset to build and calibrate the differential equations! Without this calibration, the mathematical modeling proposal would be purely theoretical.

In any case, the old double-entry matrix, a tool of the first sociologists/psychologists oriented towards quantitative research in the field, certainly cannot suffice anymore in present days.

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