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Article

An analytical method for increasing the accuracy of the value of the Newtonian constant of gravitation

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Abstract - Despite hundreds of measurements of the Newtonian constant of gravitation, its accuracy remains very low. Over the past 55 years, it has improved by only one order of magnitude - from four to five digits after the decimal point.

In this study, a new analytical method for improving the accuracy of estimating the value of the Newtonian constant of gravitation is proposed. Using the proposed method, its accuracy is increased by 7 orders of magnitude relative to the CODATA 2022 data. The method is based on the analytical estimate of the Planck mass, length, and time, with an accuracy of values that is 5 orders of magnitude higher than their accuracy according to CODATA 2022. Such a significant increase in the accuracy of the Planck mass, length, and time values was made possible by the integrated use of: 1) precision formulas for the Planck momentum; 2) representation of the speed of light in a vacuum through the Planck length and time; 3) the De Broglie principle: the moments of the Planck mass, leptons, and baryons are equal to each other; 4) high-precision characteristics of the proton. The method of analytical evaluation of Planck mass, length, and time allowed us to connect the main characteristics of the hypothetical virtual Planck particle with the main characteristics of the proton. Increasing the accuracy of the proton characteristics will entail increasing the accuracy of Planck mass, length, and time. Accordingly, the accuracy of the value of the Newtonian gravitational constant and all physical constants that can be represented through Planck mass, length, and time will be increased, which is especially important in light of the decisions of the 26^{th} General Conference on Weights and Measures.

Keywords - Physical constants; Mass, Length and Planck time; Accuracy of physical constants; Planck momentum; Newtonian constant of gravitation.

1 Introduction

There are two methods for estimating the value of the Newtonian constant of gravitation: experimental, which is the main one, and analytical. The experimental method is based on direct measurements of the value of the Newtonian constant of gravitation in laboratory conditions. Due to the influence of external factors and the complexity of accurate measurements, experimental data can vary significantly and contain errors. The analytical method for estimating the value of the Newtonian constant of gravitation consists of calculations based on known theoretical models and the laws of physics. The accuracy of this method depends on the adequacy of the mathematical models, and the accuracy of the operands, constants, and variables used to describe the gravitational interaction.

Among all fundamental physical constants, the Newtonian constant of gravitation has the lowest accuracy of value. This is due to problems in estimating its value both in experiments and in analytical models [1, 2, 3]. All formulas that represent the Newtonian constant of gravitation in explicit or implicit form contain three constants: Planck mass, length, and time. On the one hand, these constants have low accuracy of values, and on the other hand, increasing this accuracy is problematic in all aspects, since there is no real Planck particle with the above characteristics in nature. The solution to this problem and, accordingly, an increase in the accuracy of the values of Planck mass, length, and time can be realized by linking the characteristics of a virtual, hypothetical Planck particle with the characteristics of a real particle, for example, a muon, electron, proton. Problems in the experimental aspect of estimating the value of the Newtonian constant of gravitation are associated with the large-scale factor of gravitational processes, as well as the fact that gravitational interaction is the weakest of all fundamental interactions. Due to the weakness of gravitational forces, measurements of the Newtonian constant of gravitation require extremely accurate experiments using very sensitive equipment. In addition, gravitational interaction cannot be shielded, so the impact of external factors such as gravitational fields of foreign objects, temperature fluctuations, electromagnetic fields, vibrations, and other disturbances significantly complicates the measurement process. Various experiments to determine the value of the gravitational constant often lead to results that diverge by several orders of magnitude, which makes the task of increasing accuracy one of the priority problems in metrology, physics, and astrophysics. As evidence of the above, it can be noted that numerous experiments to determine the value of the Newtonian constant of gravitation from the late sixties of the last century to the present have increased its accuracy by only one order of magnitude. In 1969 the value of Newton's constant of gravity was [4]: $G = 6.6732(31) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, now according to CODATA 2022 [5] this constant has the value: $G = 6.67430(15) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$.

Experiments to measure the value of the Newtonian constant of gravitation can be divided into two groups: 1) based on the law of universal gravitation using calibration masses; 2) based on measuring the acceleration of gravity.

The first group includes the following methods:

1. The torsion pendulum method. In this method, two small masses on a torsion thread interact with two large masses. The angle of twist of the thread is measured to determine the force of interaction of the masses. In the experiment [6], based on a modification of this method, a weighted value of the Newtonian constant of gravitation was obtained: $G = (6.67554 \pm 0.00016) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, with a relative uncertainty of 24 ppm, taking into account all correlations.

2. The twist balance method. This method is a variation of the torsion pendulum method. Two large and two small masses are mounted on a rotating lever. The moment of force during the rotation of the masses is measured. In the experiment [7], using this method, the Newtonian constant of gravity has the value: $G = (6.674215 \pm 0.000092) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$.

3. Modified torsion pendulum method. This method is an improved modification of the torsion pendulum method. In this method, for example, using the time-of-swing method and the angular-acceleration-feedback method, external influences and the influence of extraneous fields are taken into account. In the experiment [8], including using this method, two independent measurements of the Newtonian constant of gravity were carried out: based on measuring the swing time of the torsion suspension and based on measuring the angular acceleration. The following values were obtained: $G = 6.674184 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$; $G = 6.674484 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, with relative standard uncertainties of 11.64 and 11.61 parts per million, respectively.

The second group includes the following methods:

1. Free-fall method. In this method, small masses are released into free fall next to large ones and their motion is recorded using interferometers or laser systems. In the experiment [9], an atomic gravity gradiometer is used to measure the differential acceleration experienced by two free-falling samples of laser-cooled rubidium atoms under the influence of nearby tungsten masses. The measurement is repeated in two different configurations of the initial masses and simulated using numerical modeling. From the evolution of the atomic wave packets and the distribution of the initial masses, the expected differential acceleration is estimated. The value for Newtonian constant of gravitation is determined by comparing experimental data and numerical modeling. The value of Newtonian constant of gravitation obtained in the experiment is: $G = 6.667 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, with estimating a statistical uncertainty of $\pm 0.011 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, and a systematic uncertainty of $\pm 0.003 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$. In the experiment [10] based on the free fall method using laser-cooled atoms and quantum interferometry, the measured value of Newtonian constant of gravitation is: $G = 6.67191(99) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, with a relative uncertainty of 150 parts per million.

2. Method of measuring the period of a pendulum. In this method, the change in the period of oscillation of a pendulum under the influence of the gravity of large masses is measured. In the experiment [11] the pendulum bobs are subjected to the gravitational pull of large masses, and the resulting small changes in the pendulum's motion are recorded using precision interferometry. The value of the Newtonian constant of gravitation measured in this experiment is: $G = (6.67234 \pm 0.00014) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$. To date, about 300 experiments have been conducted to estimate the value of the Newtonian constant of gravitation [10]. Most of the measured values are in the range: $G = (6.667 \div 6.676) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ [1]. Since 2000, measurements have mainly been grouped in the range: $G = (6.674 \div 6.675) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ [1, 12]. The accuracy of the measured values of the Newtonian constant of gravitation does not exceed 6 digits after the decimal point. This means that it is not worth expecting an increase in the accuracy of the Newtonian constant of gravitation to the accuracy of most fundamental physical constants based on experimental measurements in the near and medium term. Under these conditions, the leading method for estimating the value of the Newtonian constant of gravitation becomes the analytical method.

In [13, 14] an analytical method for increasing the accuracy of fundamental physical constants is proposed. The method is based on linking the characteristics of a hypothetical Planck particle with the characteristics of a muon based on a golden algebraic fractal. The value of the Newtonian constant of gravitation, which is calculated by this method, is: $G = 6.673045870 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$. This value is far from the range of measurements in recent years: $G = (6.674 \div 6.675) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, so there is a need to develop a new analytical method for estimating the Newtonian constant of gravitation that would not contradict modern experimental observations.

The purpose of this study is to increase the accuracy of the value of the Newtonian constant of gravitation based on an analytical method that links the mass, length and time of Planck with the main characteristics of the proton.

2 Materials and Methods

2.1 Method for analytical estimation of Planck mass, length, and time, which is based on the value of Planck momentum and proton characteristics

Based on de Broglie's formula, it is true that the moments of mass of a hypothetical Planck particle, as well as leptons and baryons, expressed in terms of their Compton wavelength (in particular, over 2π), are equal to each other [15]:

$$m_P \cdot l_P = m_e \cdot \lambda_C = m_\mu \cdot \lambda_{C\mu} = m_\tau \cdot \lambda_{C\tau} = m_{pr} \cdot \lambda_{Cpr} = m_n \cdot \lambda_{Cn}, \quad (1)$$

where respectively: $m_P, m_e, m_\mu, m_\tau, m_{pr}, m_n,$ – this is the Planck mass, as well as the masses: electron, muon, tau, proton, neutron; $l_P, \lambda_C, \lambda_{C\mu}, \lambda_{C\tau}, \lambda_{Cpr}, \lambda_{Cn},$ – this is the Planck length, as well as the Compton wavelength over 2π: electron, muon, tau, proton, neutron. In equation (1), the characteristics of the proton have the highest accuracy. Let us determine the estimate of the moment mass of the proton M_{pr}, as well as [5, 16]:

$$M_{pr} = m_{pr} \cdot \lambda_{Cpr} = 3.51767294174 \cdot 10^{-43}(1.09 \cdot 10^{-52})kg^1 m^1. \quad (2)$$

Let's calculate the value of the estimate of Planck's momentum based on various formulas. Estimation of the value of the Planck momentum P_{Pt}, which is determined through the Planck time t_P, the speed of light in vacuum c, and the reduced Planck constant $\hbar$, as well as [5, 16]:

$$P_{Pt} = \frac{\hbar}{t_P \cdot c} = 6.5247853450964(7.44 \cdot 10^{-87})kg^1 m^1 s^{-1}. \quad (3)$$

Estimation of the value of the Planck momentum P_{Pl}, which is determined through the Planck length l_P and the reduced Planck constant $\hbar$, as well as [5, 16]:

$$P_{Pl} = \frac{\hbar}{l_P} = 6.5247861113758(6.7 \cdot 10^{-70})kg^1 m^1 s^{-1}. \quad (4)$$

Estimation of the value of the Planck momentum P_{PT}, which is determined based on the Planck temperature T_P, Boltzmann constant k, and the speed of light in vacuum c, as well as [5, 16]:

$$P_{PT} = \frac{k \cdot T_P}{c} = 6.52478527(2.6 \cdot 10^{-8})kg^1 m^1 s^{-1}. \quad (5)$$

Estimation of the value of the Planck momentum P_{Pql}, which is determined through the Planck charge q_P, Planck length l_P, the speed of light in vacuum c, and the electrical constant k_e, as well as [5, 16]:

$$P_{Pql} = k_e \cdot \frac{q_P^2}{l_P \cdot c} = 6.524786111(1.2 \cdot 10^{-9})kg^1 m^1 s^{-1}. \quad (6)$$

Estimation of the value of the Planck momentum P_{Pqt}, which is determined through the Planck charge q_P, Planck time t_P, the speed of light in vacuum c, and the electric constant k_e, as well as [5, 16]:

$$P_{Pqt} = k_e \cdot \frac{q_P^2}{t_P \cdot c^2} = 6.524785345(1.2 \cdot 10^{-9})kg^1 m^1 s^{-1}. \quad (7)$$

Estimation of the value of the Planck momentum P_{PG}, which is determined through the Newtonian constant of gravity G, the reduced Planck constant $\hbar$,the speed of light in vacuum c, and the Planck mass m_P, as well as [5, 16]:

$$P_{PG} = \frac{\hbar \cdot c^2}{G \cdot m_P} = 6.5247870402(2.9 \cdot 10^{-10})kg^1 m^1 s^{-1}. \quad (8)$$

Estimation of the value of the Planck momentum P_{PMt}, which is determined through the moment of mass of the proton M_{pr}, and the Planck time t_P, as well as [5, 16]:

$$P_{PMt} = \frac{M_{pr}}{t_P} = 6.5247853450(2.02 \cdot 10^{-9})kg^1 m^1 s^{-1}. \quad (9)$$

Estimation of the value of the Planck momentum P_{PMl}, which is determined through the moment of mass of the proton M_{pr}, the speed of light in vacuum c, and the Planck length l_P, as well as [5, 16]:

$$P_{PMl} = \frac{M_{pr}}{l_P} \cdot c = 6.524786111(2.02 \cdot 10^{-9}) kg^1 m^1 s^{-1}. \quad (10)$$

Let us average the estimates of the values of Planck's momentum according to the formulas (3, 4, 5, 6, 7, 8, 9, 10) and take this averaged value as the standard P_{Pet}, as well as [16]:

$$P_{Pet} = \frac{P_{Pt} + P_{Pl} + P_{PT} + P_{Pql} + P_{Pqt} + P_{PG} + P_{PMt} + P_{PMl}}{8} = 6.52478583483(3.28 \cdot 10^{-9}) kg^1 m^1 s^{-1}. \quad (11)$$

Let us determine an estimate of the value of the Planck mass, as well as [5, 16]:

$$m_P = \frac{P_{Pet}}{c} = 2.17643428336 \cdot 10^{-8}(1.09 \cdot 10^{-17}) kg. \quad (12)$$

Let us determine an estimate for the value of the Planck length, as well as [5, 16]:

$$l_P = \frac{\hbar}{P_{Pet}} = 1.61625506752 \cdot 10^{-35}(2.87 \cdot 10^{-48}) m. \quad (13)$$

Let us determine an estimate of the value of Planck's time, as well as [5, 16]:

$$t_P = \frac{\hbar}{P_{Pet} \cdot c} = 5.39124659205 \cdot 10^{-44}(9.56 \cdot 10^{-57}) s. \quad (14)$$

Taking into account the higher accuracy of estimating the value of the Planck length using formula (13) than estimating the value of the Planck mass using formula (12), based on formulas (1, 2, 13), we will clarify the estimate of the value of the Planck mass, as well as [16]:

$$m_P = \frac{M_{pr}}{l_P} = 2.17643428468 \cdot 10^{-8}(6.74 \cdot 10^{-18}) kg. \quad (15)$$

Let's check the calculated values of Planck's length and time:

$$c = \frac{l_P}{t_P} = 299792458.0009659(0.0000752) m^1 s^{-1}, \quad (16)$$

where the uncertainty of the speed of light in a vacuum δc:

$$\delta c = \frac{l_P}{t_P} \cdot \sqrt{\left(\frac{\delta l_P}{l_P}\right)^2 + \left(\frac{\delta t_P}{t_P}\right)^2 - 2 \cdot \frac{\delta l_P \cdot \delta t_P}{l_P \cdot t_P}} = 0.0000752 m^1 s^{-1},$$

where $\delta l_P, \delta t_P$ is the uncertainty of the estimated values of the Planck length and time calculated in this study.

Estimation of the speed of light in vacuum according to CODATA 2022, as well as [5]:

$$c = \frac{l_P}{t_P} = 299792422.7919811(2.3101580) m^1 s^{-1}, \quad (17)$$

where the uncertainty δc of the speed of light in a vacuum:

$$\delta c = \frac{l_P}{t_P} \cdot \sqrt{\left(\frac{\delta l_P}{l_P}\right)^2 + \left(\frac{\delta t_P}{t_P}\right)^2 - 2 \cdot \frac{\delta l_P \cdot \delta t_P}{l_P \cdot t_P}} = 2.3101580 m^1 s^{-1},$$

where $\delta l_P, \delta t_P$ is the uncertainty of Planck length and time according to CODATA 2022.

A comparison of formulas (16) and (17) shows the significant effectiveness of the method proposed in this study for increasing the accuracy of Planck mass, length, and time.

2.2 Improving the accuracy of the Newtonian constant of gravitation

Using various physical formulas, we calculate estimates of the Newtonian constant of gravitation, and then average the resulting values, as well as [5, 16]:

$$G_1 = \frac{l_P^3}{m_P \cdot t_P^2} = 6.67430035599 \cdot 10^{-11}(8.32 \cdot 10^{-26}) m^3 kg^{-1} s^{-2}. \quad (18)$$

$$G_2 = \frac{l_P}{m_P} \cdot c^2 = 6.67430035595 \cdot 10^{-11}(1.19 \cdot 10^{-23}) m^3 kg^{-1} s^{-2}. \quad (19)$$

$$G_3 = \frac{\hbar \cdot c}{m_P^2} = 6.67430035187 \cdot 10^{-11}(5.38 \cdot 10^{-31}) m^3 kg^{-1} s^{-2}. \quad (20)$$

$$G_4 = \frac{l_P^2 \cdot c^3}{\hbar} = 6.67430036002044 \cdot 10^{-11}(exact) m^3 kg^{-1} s^{-2}. \quad (21)$$

$$G_5 = \frac{t_P^2 \cdot c^5}{\hbar} = 6.67430035997743 \cdot 10^{-11}(exact) m^3 kg^{-1} s^{-2}. \quad (22)$$

Using formulas (18 - 22), we calculate the estimate of the average value of the Newtonian constant of gravitation, as well as [16]:

$$G = \frac{G_1 + G_2 + G_3 + G_4 + G_5}{5} = 6.67430035676157 \cdot 10^{-11}(2.38 \cdot 10^{-24}) m^3 kg^{-1} s^{-2}. \quad (23)$$

3 Result

The accuracy of the estimate of the Newtonian constant of gravity according to formula (23) is 7 orders of magnitude higher than the accuracy of the Newtonian constant of gravitation according to CODATA 2022.

4 Conclusions

1. A new analytical method has been developed to improve the accuracy of the Newtonian constant of gravitation, based on averaging the values of the Planck momentum, calculated using high-precision formulas, the De Broglie principle, and the characteristics of the proton.
2. The resulting estimate of Newtonian constant of gravitation has an accuracy that is 7 orders of magnitude better than the current CODATA 2022 value.
3. Improving the accuracy of the Newtonian constant of gravitation allows: to improve the consistency of experimental data and astrophysical measurements with theoretical predictions, to increase the accuracy of calculations in astrophysical and cosmological models, to reduce the uncertainty in gravitational measurements and calculations.
4. The method can be applied to improve the accuracy of other multidimensional physical constants, which can be represented through the Planck mass, length, and time.

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