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Predicting Super Massive Black Hole Collisions Using LISA

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Abstract – LISA (Laser Interferometer Space Antenna) is due for launch in the 2030s. Its goal is to observe gravitational waves in the 10^{-4} to 10^{-1} Hz band. This frequency band contains signals from colliding Super Massive Black Holes, objects with masses in the range of millions, even billions, that of our own suns mass. These SMBHs are thought to lie at the heart of most, if not all galaxies. By understanding the physics of the underlying processes, and what LISA 'sees', we can predict when these mergers will occur. This would allow us to observe the merger directly in the EM spectrum, observing the light emitted from the merging accretion discs. This could yield a potentially vast amount of information about the composition and formation of these huge objects. In this project we explore some of the potential variations of the signals detected, and show that we can detect the merger several days prior to it occurring.

Keywords – Black Hole, Super Massive Black Hole, LISA, Gravitational waves.

1. Introduction

Gravitational wave astronomy is a relatively new field, beginning with the prediction of gravitational waves by Albert Einstein's Theory of General Relativity in 1916 [1]. Direct evidence for gravitational waves was not detected until 2015, a century later, when LIGO (Laser Interferometer Gravitational Wave Observatory) detected the merger of 2 Black Holes (GW150914) approximately 1.337 $\times 10^{9}$ m Light Years (410 Mpc) away, with a respective mass of 29x and 36x the mass or our own sun ($M_\odot = 1.989 \times 10^{30}$ kg) [2]. This type of merger is known as a Compact Binary Coalescence (CBC).

LISA (Laser Interferometer Space Antenna) is a space-based interferometer due for launch in the 2030s by the ESA (European Space Agency) [3]. Its goal is to detect gravitational waves in the mHz range, which covers the mergers of the Super Massive Black Holes that are thought to reside at the heart of most galaxies. Very little is known about these extreme objects, but over vast timescales, they can become gravitationally bound, and begin to spiral into each other, eventually coalescing and merging [4].

This project shows LISAs ability to detect gravitational waves from these mergers, and shows how the physics of the merger could allow us to directly observe a merger in almost real time, potentially yielding huge clues as to how these giants form.

2. Theory

2.1 Black Holes

Black Holes are regions of space so dense that not even light can escape. Although their location can be inferred from the gravitational effect on other objects, if looked at directly they would be invisible to our telescopes (although the light from objects behind the black hole would be warped via gravitational lensing - another prediction of general relativity) [5].

However, as black holes gravitationally attract matter, some of that matter would enter orbit in what is known as the accretion disc. When this matter experiences the extreme gravitational acceleration and tidal forces near the event horizon, it emits light via friction which can be detected. In the context of a collision, the merger of these accretion discs would emit X-Rays and Gamma Rays, which if analyzed, could yield clues as to how these objects form [6].

The Super Massive Back Holes at the centers of galaxies are thought to initially form as Stellar Mass Black Holes, but then by merging with other objects and absorbing their mass, they can grow to the sizes we expect today. However, the exact mechanism of this process is unclear, but the remnants of such objects found in the accretion discs may shed light on this process. By observing the merger of 2 such SMBHs, we could detect the composition of this material, and thus infer information about their formation [7].

2.2 Gravitational Waves

As these black holes orbit into each other, they release energy in the form of Gravitational Radiation. As explained by general relativity, this energy warps space itself, leading to variations in a known length [8]. This is given by the strain (h), a dimensionless quantity that LISA is measuring via interference. As the orbit inspirals due to this energy loss, the gravitational wave produced increases in frequency and amplitude, to a maximum value f_{max}. The frequency corresponds to the period of the orbit (as orbital frequency) by a factor of 2. The amplitude is related to the masses of the object, where a heavier mass will produce a higher strength signal [9].

As the frequency is directly related to the orbital period, it is in turn dependent on the distance between the two objects as approximated by Kepler's 3rd law. Although this relation breaks down shortly before collision as the physics of GR supersedes Newtonian, it is suitable for our purposes. The frequency just before collision, when the objects are very close together and the orbital period very small, is very high compared to when the objects are far apart and take a longer time to orbit each other [10].

It is then easy to see that the earlier in the process of coalescence, the lower the frequency range of the signal generated. Thus, considering the waveform at a value below f_{max}, corresponds to seeing it earlier in time. This is also why different mass ranges occupy different frequency bands [11].

3. Methodology

To model the signals, we used the Python programming language with modules from the PyCBC package. This package contains a variety of tools for interacting with gravitational wave data, and was used in the detection of the merger GW150914 [12]. The package contains a number of waveform templates, and there are differences between how the waveforms are generated, each with their pros and cons. An equation for the strain is given as [13]:

$$h(t) = \frac{4M\eta v^2}{r} e^{-i(2\omega_{orb}t+\Phi_0)} \quad [1]$$

Much of this comes from General Relativity, and is beyond the scope of this paper, but we would draw attention to the dependence on distance (r), and the frequency (ω_{orb}). Mass also affects the strain, but here the mass is encoded within M, the 4D spacetime manifold.

We used the template "SEOBNRv4_opt", because it is generated in the time domain. This may not be the fastest template to use, but it does remove some of the complications with modelling in the frequency domain.

We also needed to select a sample rate and a lower frequency cutoff. These are purely modelling choices and are not changing the physics of the merger, but they will affect the modelled signal, and hence the SNR calculated. It is important to keep these values consistent as we change the waveform.

For the sample rate we used a value of 16. This value was a balance between numerical accuracy and computational efficiency; the higher the number, the more accurate the simulation will be, but the program will have to perform more calculations.

The frequency cutoff is the value at which the computation begins modelling. For our purposes, we wish to consider this as the point that LISA could begin detecting 'something'. Thus, we chose a value of 10^{-5} Hz, as this is the lower end of LISAs sensitivity [14]. There is little value to be gained from running a computationally intensive simulation for parts of the waveform we would be unable to see.

We then generated an initial signal based off this template, such that it will generate a complete signal, with merger and ringdown. There are a range of parameters, but we will only be focusing on mass and distance. For the mass, we used equivalent mass black holes at 4.1 $\times 10^6$ $M_\odot$, the mass of Saggitarius A*: the SMBH at the center of the Milky

Way [15]. For distance, we took the edge of the observable universe as being 4.4 x10^{26}m, which we converted into 14,255 Mpc [16]. Note that the code takes units of Solar Masses and Mpc, while the equations used here are in SI units.

We then vary the initial parameters to explore the effect of distance, mass, and mass ratio.

When varying the mass, we must consider the chirp mass. The chirp mass of a compact binary system determines the rate at which energy is lost from the system, and is thus used to more efficiently model the signal. Our model assumes a linear energy loss with increased mass, which is a simplification. The true value is given by the chirp mass, which is also more convenient to measure in practice than the individual component masses. The chirp mass is given by [17]:

$$M_{chirp} = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}} \quad [2]$$

The time remaining to merger is denoted by τ, and it follows that increasing this will result in a longer waveform given by [18]:

$$\tau(f) = \frac{5}{256}(\pi f)^{\frac{-8}{3}}(M_{chirp})^{\frac{-5}{3}} \quad [3]$$

We developed an iterative process using Eq. 3 that keeps the Low Frequency Cutoff and Sample Rate consistent relative to the length of the waveform. Details of this are given in the Appendix.

To quantify how well LISA can see these signals, we used the 'sigma' module to return the SNR (Signal to Noise Ratio). The 'sigma' process is derived from the matched filter, a process which follows [19]:

$$SNR = 4\Re\int_0^{\infty} \frac{\tilde{s}(f)\cdot\tilde{h}(f)}{S_h(f)}df \quad [4]$$

This is known to be the most efficient way of detecting a signal in modelled noise, with the caveat that the signal being searched for, and the noise, must be known. An explanation of this process is given in the Appendix [20].

The noise, $S_h(f)$, here is modelled by LISAs sensitivity curve or ASD (Amplitude Spectral Density). This is formed from modelling the expected sources of gravitational radiation, over the sensitivity range [21].

The sigma function takes as input: the time series from the signal, the ASD, and also a low and high frequency cutoff. Initially we set low frequency cutoff to be the same as in the modelled signal, and high frequency cutoff left blank. If we had selected a value (as we shall later), the computation would run only until the signal reached this frequency. As it was blank, the signal is allowed to run until merger/completion. This is how we explore early detection later.

The sigma function outputs the cross and plus polarizations of the waveform. To account for discrepancies when they begin modelling, we add them in quadrature and half to find the 'average' value of the waveform at that point.

In terms of detectability, we have used a value of SNR = 8 as the detection threshold. This means that when signal has achieved an SNR of 8, we can say confidently that LISA would be able to detect this signal. This is in line with previous work that used an SNR of 8 [22]. It is important to note that the ultimate aim of this project, locating a merger on the sky, would require a higher SNR than this, but as such a calculation is beyond the scope of this paper, we have left this as a topic to be explored.

4. Results

4.1 Distance

The distance to the edge of the observable universe is 4.4 x10^{26}m, which we converted into 14255 Mpc. Placing this value in our code yields a result of SNR = 3222. This signal is shown in Fig 1, and will be described as out 'initial signal'. These results suggest that LISA will in fact be able to see these collisions from anywhere within the observable universe. The actual variation with distance is shown in Fig 2 and Fig 3. The distance variation is well established to be inversely linear, as seen in Eq. 1.

We can also plot this in the frequency domain to see the interaction with the ASD. This is shown in Fig 3. Note how the waveform moves directly down as the distance increases. This reflects the linear relationship between distance and SNR.

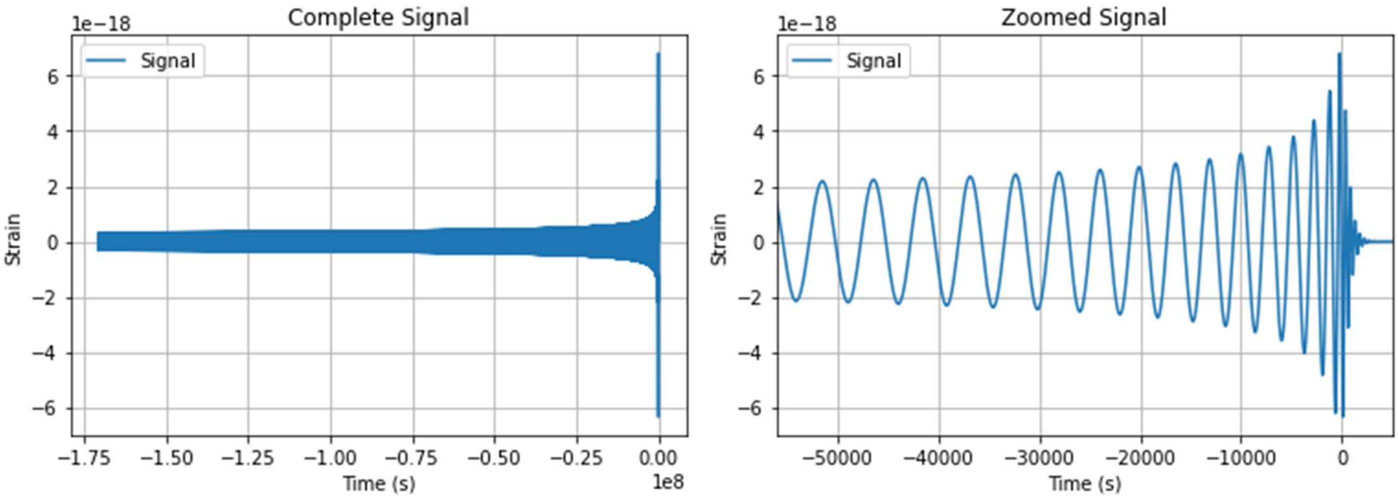

Figure 1. A Binary Black Hole merger at 14,255 Mpc. Equal masses at 4.1 x10^6 $M_\odot$. The SNR for this was 3222.8149

4.2 Mass

We then want to see how the mass of the system would affect the SNR. Keeping the masses identical, we then varied the mass of the objects, and recorded the
returned SNR for that system. This is shown in Fig 4.

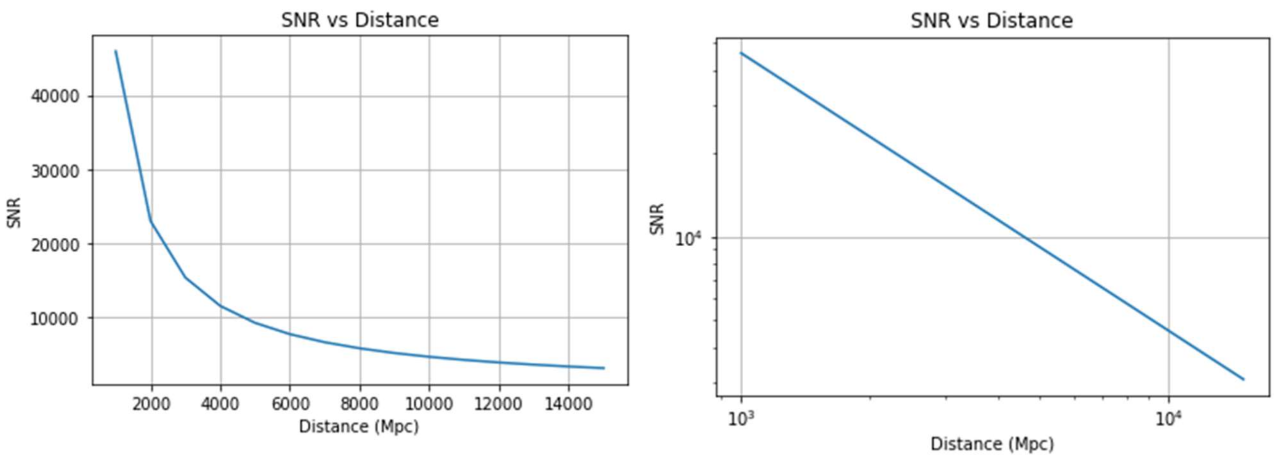

Figure 2. How the calculated SNR for a BBH merger with equivalent masses at 4.1 x10^6 $M_\odot$ changes with distance.

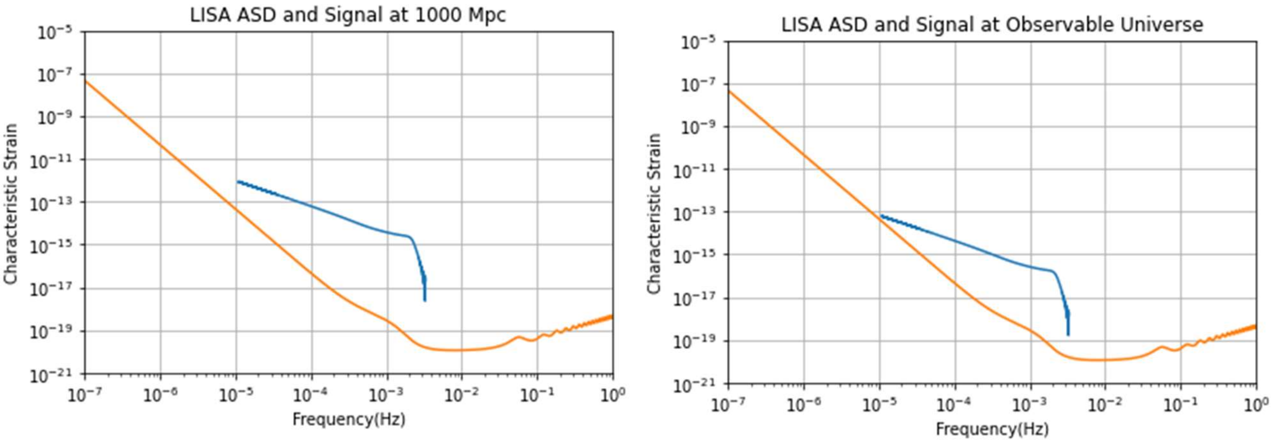

Figure 3. The effect of distance in the frequency domain. Both mergers have equivalent masses (m1 = m2 = 4.1 x10^6 M⊙), only the distance has been changed, from 1000Mpc to 14,255 Mpc.

This result was unexpected. I expected a more linear response, with the SNR generally increasing with the mass. The model presented here suggests that in fact there is a peak mass, at which the signal will be loudest at a given distance, before it appears to asymptote away as the mass continues to increase.

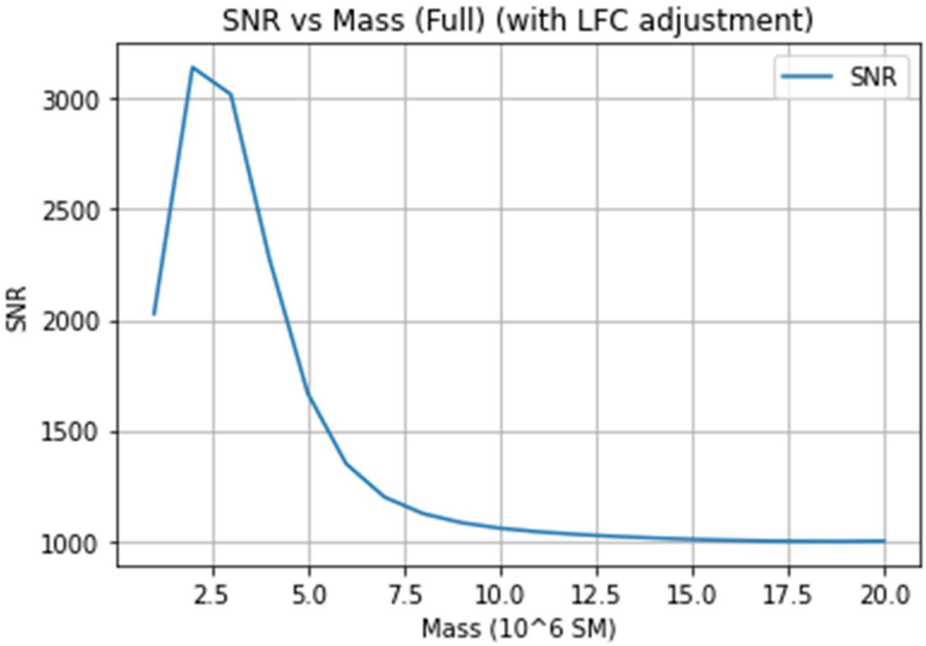

Figure 4. SNR vs Mass at a distance of 14,255 Mpc.

The reason for this is the ASD. We plotted the Fourier transform (frequency domain) of the signal on the same plot as the ASD to investigate this. The lower the ASD is, the higher the sensitivity at that frequency. We can then see from Fig. 5 that the ASD has a peak sensitivity at around 10^{-2} Hz. For masses around 2.5 x10^6 M⊙, the signal sits almost exactly in this peak (trough?) sensitivity range. Increasing the masses causes it to shift slightly up, and to the left. It is this interaction with the ASD that causes the variation in the SNR vs Mass plot. Further investigation of this would involve extending the masses up to the billions of M⊙, to see if there is an upper limit to LISAs detection.

It does however seem that there is a peak mass sensitivity, at least for the case of 2 identical masses, at around m = 2.5x10^6 M$_\odot$.

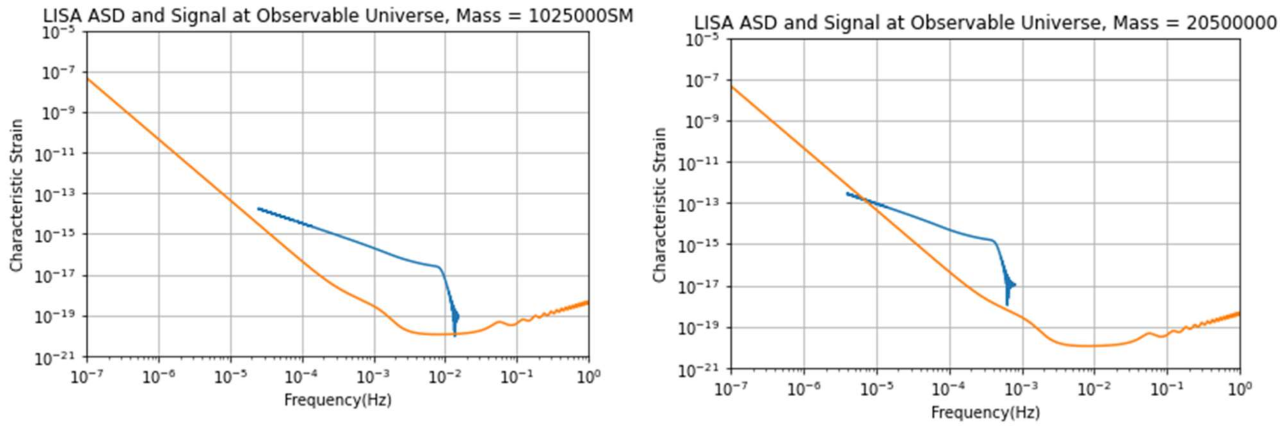

Figure 5. Here we see the effect of mass in the frequency domain, and how it affects the SNR. Both these mergers were modelled at 14,255 Mpc.

4.3 Mass Ratio

We then wanted to see how the mass ratio of the objects would affect the SNR. It is unlikely that an observed merger would have equivalent masses, and so this gives us an indication of a more ‘real world’ scenario. Keeping the total mass (m1 = 4.1 x10^6 M$_\odot$, m2 = 4.1 x10^6 M$_\odot$, m_{tot} = 8.2 x10^6 M$_\odot$) of the bodies from the initial signal constant, we varied the proportion of this total that each body made up.

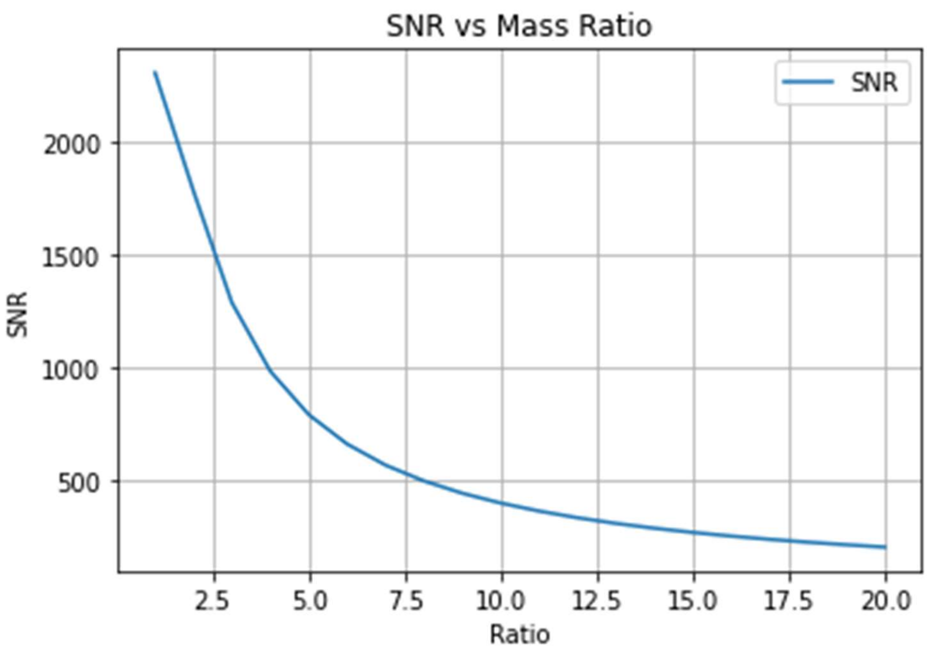

Figure 6. SNR vs Mass Ratio at a distance of 14,255 Mpc.

This modelling was chosen for simplicity, but further exploration of this would want to consider keeping the chirp mass constant.
The results of this process are shown in Fig. 6. The main takeaway is that LISA is most able to detect objects when their masses are equal. We can understand this by considering orbital mechanics, and that the gravitational radiation arises from the movement of these massive objects.

As an example, although it is fair to say that the Earth orbits the Sun, it is more accurate to say that both the Earth and the Sun orbit a point very close to the Suns centre of mass, but not exactly. This is because both bodies exert a gravitational attraction on each other, rather than just the Earth being attracted to the Sun. Indeed, Jupiter famously orbits a point outside the Sun [https://spaceplace.nasa.gov/barycenter/en/]. The key point is that while the small mass object experiences a considerable acceleration, the high mass object does not.

Extending this to the black holes we are considering, it is easy then to see that as the mass ratio increases, the heavier object (that would produce the most gravitational radiation) moves less, while the lighter object will move more, but due to its decreased mass, will produce less gravitational radiation. At extreme ratios, this system becomes better modelled by a single high mass object, which would not produce large amounts of gravitational radiation. This is another area where using chirp mass can simplify the equations.

4.4 Early Detection

Here, we intend to look at the point at which the signal becomes detectable. We use the previously defined threshold of SNR = 8 for a detection.

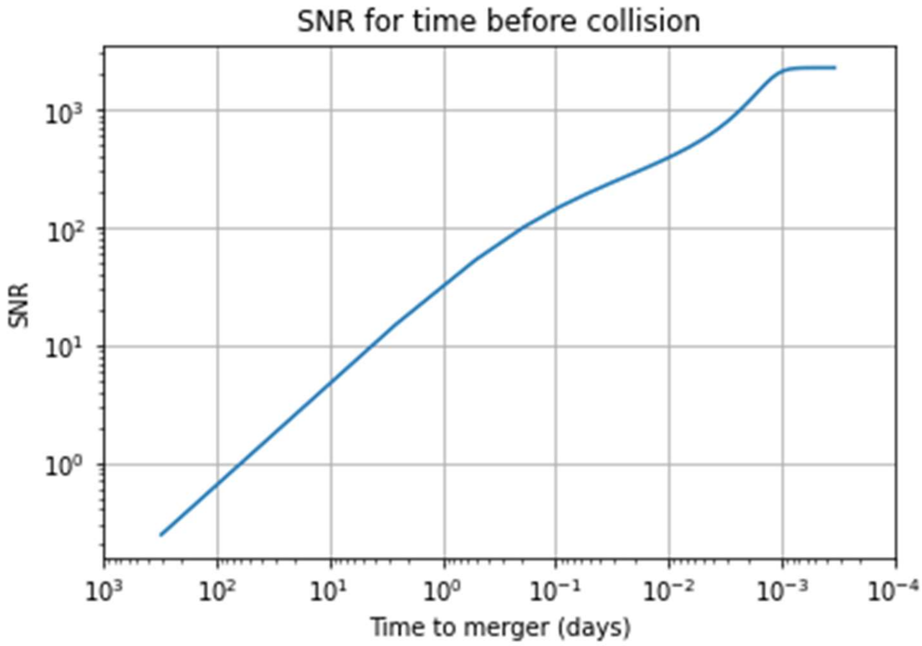

Figure 7. The SNR over time for a merger at 14,255 Mpc with equivalent masses (m1 = m2 = 4.1 x$10^6 M_{\odot}$)

We now use the high frequency cutoff mentioned earlier. As the frequency increases with time, stopping the waveform generation at a frequency corresponds to stopping the waveform generation earlier in time. By setting this value to a frequency below f_{max}, the waveform generation will stop when it reaches this frequency. We can then run the sigma process to obtain the SNR for this reduced/early waveform. By combining this with an iterative process, we can find the point at which the signal reaches the threshold of SNR = 8.

Due to the intensive calculations, I have run a few variations of this code. Initially we look at a high sample rate (inaccurate but computationally efficient) for the entire signal. This is shown in Fig. 7. However, since this is a very high SNR, we do need to ‘zoom in’ a little.

We can get a more accurate result by running the code using a lower HFC iterative increase. This increases the

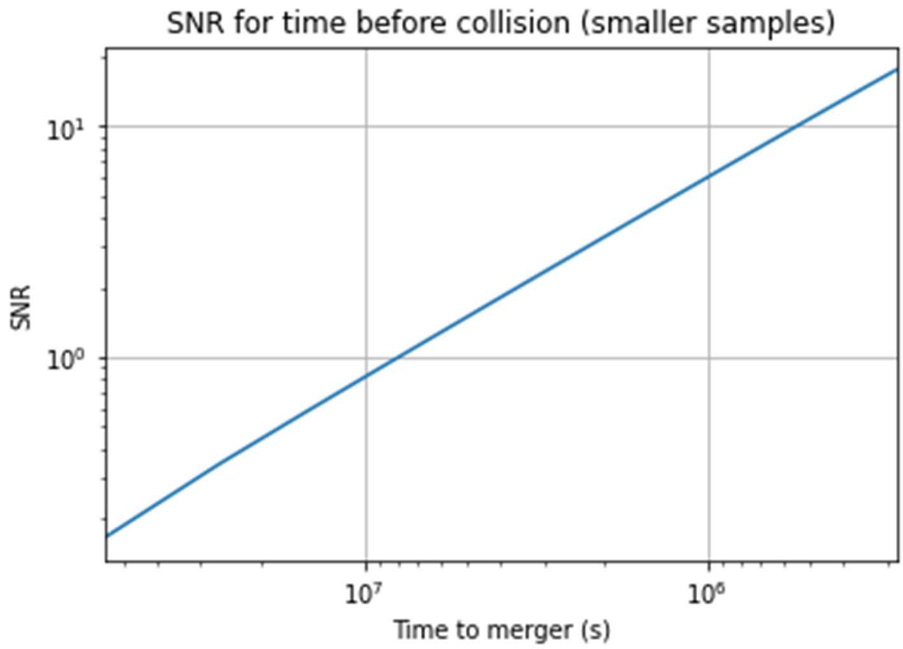

Figure 8. The SNR over time for a merger at 14,255 Mpc with equivalent masses (m1 = m2 = 4.1 x10^6 M⊙).

computational cost, but we can limit ourselves to a reasonably low HFC. This is shown in Fig. 8. From this data, we found that SNR = 8.929 at the timestamp t_c - 414740. This suggests that LISA could achieve advance warning for our initial signal around 5 days before the merger occurred.

4.5 Errors

It is important to consider the applicability of these results. Our model has not considered many of the real-world variables that would occur in a physical merger. Not only the modelling variables, but also calibration errors arising from LISAs interference equipment. There are also uncertainties in how the waveform is generated, due to the approximations used to emulate the General Relativity equations.

LISAs sensitivity curve is an estimate of the noise. There is a statistical error of +/-1 in this estimate. The goal is for the systematic errors above to be cumulatively lower than the statistical error, but this is difficult to quantify at present due to the nature of the project.

5. Discussion

The aim of the project was to investigate the feasibility of early detection. We have been able to demonstrate that the concept is sound in principle, with a cautious estimate of around 5 days. We have shown how varying the parameters can affect this result. Due to the complexity of modelling these signals, further investigation would be needed to give a definitive result.

We would have liked to see how this estimate would change with the mass ratio, but were unable to complete this part of the project due to coding issues. A partially modelled code can be found in the Appendix. Additional improvements could also be made to efficiency, and extracting values.

The unmodelled variations do affect the confidence of this specific result, although I do consider it to be a reasonable estimate. This value does not consider orbital precession, inclination, or eccentricity, all of which would have a negative effect on the SNR, and thus would reduce the early warning time. Neither does it account for redshift. Although this is likely to have a positive effect on the SNR, it is unclear how well this would translate into an earlier detection.

Conversely, as we have modelled this merger at the edge of the observable universe, by definition any merger observed would be closer, and thus have a higher SNR. Indeed, the interaction of increased distance against the increased redshift that comes with it would be an interesting direction to take this.

A significant consideration on the viability of early detection is how long it would take to orient the telescopes correctly. Consideration must also be given to the time taken to recognize the signal.

Perhaps the key area for further work would be the SNR required to locate the merger on the sky. The main motivation for this early detection is the idea of observing the merger in real time with as many telescopes as possible, in order to collect the most amount of data. Although an SNR of 8 is suitable to calculate the time of the merger, its spatial location is much more precise, and would require a much higher SNR. This would obviously have a significant impact on the early warning time.

Another area would include extending the masses up to the 10^9 $M_\odot$ range. Although it is unlikely LISA would observe a merger in this mass range over the course of its lifetime, it would be interesting to see if there is an upper limit to this detection range.

6. Conclusion

We set out to obtain an estimate for how much notice LISA could give for an imminent merger. We described the merger of 2 Super Massive Black Holes, and modelled a signal based on the described parameters. We have looked at range of variables involved in the generation of a waveform template, and shown how they affect the modelled signal. We have then shown how changing these variables affects the loudness of the signal, and in turn, how this affects the advance warning of an impending merger. From this, we estimated an advance warning of around 5 days for the merger of 2 identical black holes with masses 4.1 x10^6 $M_\odot$, if they merge at the edge of the universe.

We have discussed the interpretation of these results, and the important caveats they come with. This does not account for real world practicalities, such as signal processing time, or the time needed to adjust observations. We have also drawn attention to the complex nature of these mergers, and the need to analyze further variables before reaching a firm conclusion on the validity of these estimates.

In conclusion, it is clear that gravitational wave astronomy is a complex and exciting area of research. This project helps set the stage for further analysis and preparation for LISAs launch in the 2030s.

Appendix

Code

The code used in the project can be found as a Jupyter notebook here (https://github.com/up915989/Predicting-Super-Massive-Black-Hole-Collisions-Using-LISA)

We did manage to partially run the early detection for the mass ratio, and confirmed that the mass ratio would have a negative effect on this early detection parameter. This code took a very long time to run, and there was not enough time to run the tweaks needed. For example, it is unclear why the values at 14 and 16 are identical.
I havencluded the partial work here for completeness, and to demonstrate that the hypothesis was sound. It is also included in the GitHub upload. It is also included in the GitHub upload.

Black Hole Formation

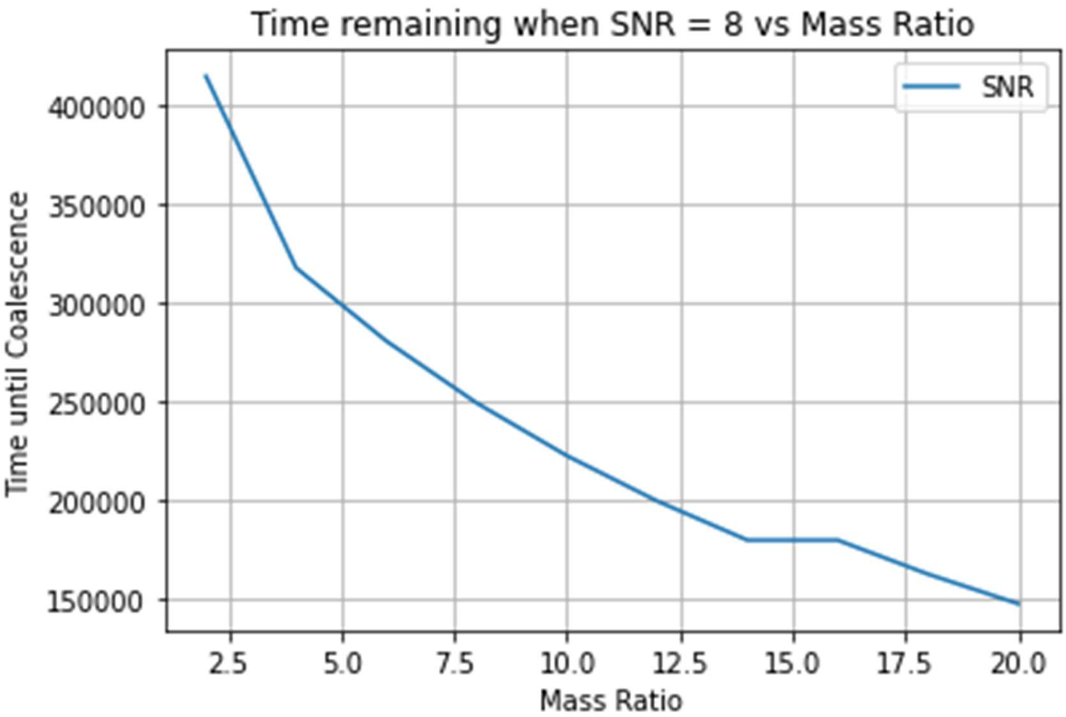

Figure 9. The time left after an SNR of 8 is achieved, and how this varies with mass ratio.

Most black holes are largely expected to form via stellar collapse, although alternative theories such as primordial formation do exist. As a star nears the end of its life, its fate is decided by the outcome of a struggle between two opposing forces: The gravitational attraction from the mass of the star is countered by the pressure produced via fusion. For stars below a certain mass the gravitational attraction is unable to overcome the pressure from the ongoing fusion reactions, and the star explodes in a highly energetic process known as a Supernova.

For stars above this mass limit, gravity wins out. The outer layers of the star are shed due to the pressure from fusion, but the core collapses under its own gravity. This runaway collapse compresses the matter to an infinitesimally small region, with infinite density, known as the singularity. This is surrounded by an event horizon, which is the point at which the gravitational pull is so strong, an object would need to be travelling faster than the speed of light to escape. It is therefore (supposedly) impossible to observe a singularity directly.

Other theories of formation include a chain reaction of sorts, where early stars were clustered so close they merged themselves and became an Intermediate Mass Black Hole. These objects then encountered other IMBHs and merged with them to become SMBHs.

Redshift

We have also not considered the effect of redshift. Where we are observing these mergers at the edge of the universe, redshift is a consideration. However, due to the nature of the waves, this redshift will not inherently affect the detectability of the signal; indeed the effect would only increase it. This is because LISA only directly measures the amplitude of the signal, which is not affected by redshift. It would however affect the frequency, and this would cause the waveform to appear as though it had been generated by higher mass objects than would actually be. Objects will inherently appear heavier at higher redshifts.

Modelling

There are potentially 15 parameters to vary when considering waveforms, a full discussion of which is beyond the scope of this project, so we have made some assumptions to simplify this. We are considering a merger between 2 black holes. Their orbit is circular (has 0 eccentricity), and the angular momentum of the black holes themselves is oriented perpendicular to the plane of the orbit. If the angular momentum was not perpendicular to the plane, the orbit would precess and the amplitude of the waveform would vary, complicating our model. These are areas that would be explored in a continuation of this project.

Matched Filter

In practice, the matched filter functions by taking the data (a time series composed of the hidden signal and noise denoted by $\tilde{s}(t)$), and transforming that data into the frequency domain via Fourier transform giving $\tilde{s}(f)$. It then does the same with the modelled signal template $\tilde{h}(t) \rightarrow \tilde{h}(f)$, where h is the amplitude of the signal, and then sequentially comparing them in the frequency domain against the noise $S_h(f)$. The function then returns a sum of these values, which is a representation of how well the signal in the noise matches the signal template.

LFC Adjustment

We want to keep $\tau(f)$, and thus the waveform length, consistent. If we change the mass by x, the chirp mass will also change by x (Eq. 2). This results in a change to $\tau(f)$ of $x^{-5/3}$. We must therefore adjust the frequency cutoff by a value $(x^{-5/3})^{3/8} = x^{-5/8}$. This is the LFC (Low Frequency Cutoff) adjustment process referenced in the results.

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References

[1] LIGO Lab | Caltech. n.d. *What are Gravitational Waves?*. [online] Available at: <https://www.ligo.caltech.edu/page/what-are-gw>
[2] Abbott, B. et al. GW150914: First results from the search for binary black hole coalescence with Advanced LIGO. *Physical Review D*, 93(12).
[3] Sci.esa.int. 2017. *ESA Science & Technology - Gravitational wave mission selected, planet-hunting mission moves forward*. [online] Available at: <http://sci.esa.int/cosmic-vision/59243-gravitational-wave-mission-selected-planet-hunting-mission-moves-forward/>.
[4] Science.nasa.gov. n.d. *Black Holes | Science Mission Directorate*. [online] Available at: https://science.nasa.gov/astrophysics/focus-areas/black-holes
[5] Bozza, V., 2009. Gravitational Lensing by Black Holes. *General Relativity and Gravitation*, 42(9).
[6] Rosswog, S., 2007. Fallback accretion in the aftermath of a compact binary merger. *Monthly Notices of the Royal Astronomical Society: Letters*, 376(1), pp.L48-L51.
[7] Bromm, V. and Yoshida, N., 2011. The First Galaxies. *Annual Review of Astronomy and Astrophysics*, 49(1), pp.373-407.
[8] Einstein, A. and Rosen, N., 1937. On gravitational waves. *Journal of the Franklin Institute*, 223(1), pp.43-54.
[9] LIGO Scientific and VIRGO Collaborations. 2016. *The basic physics of the binary black hole merger GW150914*. [online] Available at: https://arxiv.org/pdf/1608.01940.pdf
[10] Abbott, B., 2016. *Observation of Gravitational Waves from a Binary Black Hole Merger*. [online] Journals.aps.org. Available at: https://journals.aps.org/prl/pdf/10.1103/ PhysRevLett.116.061102
[11] Prince, T., 2009. The Promise of Low-Frequency Gravitational Wave Astronomy. Available at: https://arxiv.org/abs/0903.0103v1
[12] Nitz, A., Harry, I., Brown, D., Biwer, C., Willis, J., Canton, T., Capano, C., Dent, T., Pekowsky, L., Williamson, A., Davies, G., De, S., Cabero, M., Machenschalk, B., Kumar, P., Macleod, D., Reyes, S., Pannarale, F., Massinger, T., Kumar, S., Tápai, M., Singer, L., Khan, S., Fairhurst, S., Nielsen, A., Singh, S., Gadre, B. and Dorrington, I., 2021. *gwastro/pycbc: PyCBC Release 1.18.1*. [online] Zenodo. Available at: https://zenodo.org/record/4849433#.YLkr6i2ZO8o
[13] Schmidt, P., 2014. *Studying and Modelling the Complete Gravitational-Wave Signal from Precessing Black Hole Binaries*. [online] Orca.cf.ac.uk. Available at: https://orca.cf.ac.uk/64062/1/2014schmidtpphd.pdf
[14] Lisa.nasa.gov. n.d. *LISA - Laser Interferometer Space Antenna -NASA Home Page*. [online] Available at: https://lisa.nasa.gov
[15] Abuter, R., Amorim, A., Bauböck, M., Berger, J., Bonnet, H., Brandner, W., Clénet, Y., Coudé du Foresto, V., de Zeeuw, P., Dexter, J., Duvert, G., Eckart, A., Eisenhauer, F., Förster Schreiber, N., Garcia, P., Gao, F., Gendron, E., Genzel, R., Gerhard, O., Gillessen, S., Habibi, M., Haubois, X., Henning, T., Hippler, S., Horrobin, M., Jiménez-Rosales, A., Jocou, L., Kervella, P., Lacour, S., Lapeyrère, V., Le Bouquin, J., Léna, P., Ott, T., Paumard, T., Perraut, K., Perrin, G., Pfuhl, O., Rabien, S., Rodriguez Coira, G., Rousset, G., Scheithauer, S., Sternberg, A., Straub, O., Straubmeier, C.,

Sturm, E., Tacconi, L., Vincent, F., von Fellenberg, S., Waisberg, I., Widmann, F., Wieprecht, E., Wiezorrek, E., Woillez, J. and Yazici, S., 2019. A geometric distance measurement to the Galactic center black hole with 0.3% uncertainty. *Astronomy & Astrophysics*, 625, p.L10.

[16] Vlahovic, B., 2021. A New Cosmological Model of the Universe and New Interpretation for the Cosmic Microwave Background and Type Ia Supernovae Redshifts. [online] Arxiv.org. Available at: https://arxiv.org/vc/arxiv/papers/1005/1005.4387v1.pdf

[17] Cutler, C. and Flanagan, É., 1994. Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral waveform?. *Physical Review D*, 49(6), pp.2658-2697.

[18] Harry, I., 2011. Can we hear black holes collide?. [online] p.36. Available at: http://orca.cf.ac.uk/55138/1/U585527.pdf

[19] Brown, D., 2007. *Searching for Gravitational Radiation from Binary Black Hole MACHOs in the Galactic Halo*. [online] arXiv.org. Available at: <https://arxiv.org/abs/0705.1514v1> [Accessed 3 June 2021].

[20] Schutz, B., 1999. Gravitational wave astronomy. *Classical and Quantum Gravity*, 16(12A), pp.A131-A156.

[21] Robson, T., Cornish, N. and Liu, C., 2019. The construction and use of LISA sensitivity curves. *Classical and Quantum Gravity*, 36(10), p.105011.

[22] Abbott, B. et al, 2017. Search for intermediate mass black hole binaries in the first observing run of Advanced LIGO. *Physical Review D*, 96(2).