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Modelling the Sea Surface of Cardigan Bay

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Abstract - The aim of this project was to model the sea surface of Cardigan Bay, to be able to expand upon QinetiQ's current wave models, used within their radar assurance activities. The sea surface can cause unwanted detections, which is referred to as clutter. The sea is not the only source of clutter such as litter and birds, have an impact on the returns detected by the radar. This project explored two different approaches, modelling the propagation of waves and modelling the distribution of energy across the sea surface. The first approach explored two open source models, WAVEWATCH III and SWAN, and after having technical errors, two real world, existing examples of where these models were implemented were researched. This research showed a lot of promise, and would be worth expanding upon. The second approach looked into two recognised equations for calculating the energy density of the waves, and then calculating the significant wave height. Overall, modelling the propagation of waves would produce more representative results, particularly SWAN which was a model designed for coastal waters, however to set up the wave models fell outside of the scope of this project.

Keywords - Wind Waves; Source Functions; Pierson-Moskowitz; JONSWAP; Significant Wave Height

1 Introduction

1.1 Project Aim

The aim of this project was to model the sea surface of Cardigan Bay, to be able to expand upon QinetiQ's current wave models, used within their radar assurance activities.

1.2 Relevance of the Sea Surface on Radar

Radar is prevalent in today's society, ranging from military applications to the tracker inside of a mobile phone. Part of using radar within society is ensuring that the radar is performing to its expected capability. This is done by characterising the radar within a model that captures the key parameters of the radar. Then comparing the results with the measured real world data, to ensure it's still effective. QinetiQ follows this process regularly to assure sea surveillance radars around the UK.

QinetiQ has a suite of radars that they test on the range at Aberporth, using radar models and real world data. QinetiQ has supported this research project to expand upon their current wave models, that are integrated into their radar models, to improve their radar assurance activities.

The sea surface can cause unwanted detections, which is referred to as clutter. The sea is not the only source of clutter such as litter and birds, have an impact on the returns detected by the radar. This results in smaller objects of interest, such as kayaks and pleasure craft, being harder to detect in the ocean. If there is too much clutter, the radar will have too many readings to identify any objects of interest. However, the sea is the main source of clutter, therefore understanding the impact of the sea surface would improve the accuracy of radar models.

Many radars utilise the Doppler effect. The Doppler effect is the change in frequency detected by an observer from a source that is emitting waves (e.g sound, light and radio frequency (RF)) while moving relative to the observer. If the source is moving towards the observer, then there is a blue shift, where the wave is becoming compressed and thus the frequency is increasing. Conversely, if the source is moving away from the observer, then there is a red shift, the wave is stretched, and the frequency decreases. This phenomenon is used in radar to find objects by using the received Doppler frequency to determine the difference in velocities which allows the separation of the object and the waves.

This process of determining the difference between clutter and an object of interest is used by QinetiQ for both their radar assurance activities and range safety protocol. For the range at Aberporth, who fire explosive ordnance into the sea, it is essential to ensure that members of the public are not near the range during this testing. This is done by using radar to monitor for anomalous returns within the firing area. To test the sea surface models, that will be detailed in this report, it is essential to compare the results with real world data to determine the effectiveness of the model. To support this endeavour, the site at Aberporth has supplied real world data, which will henceforth be referred to as the ground truth.

1.3 Research Ambitions

Whilst there are a multitude of factors that affects the sea surface, this project has focused primarily on wind waves; waves that are propagated by the transfer of energy from the wind to the sea surface via friction. Considering factors like the humidity and the mean sea level (MSL) pressure are significant for developing a more representative model, this project focuses on the impact of the wind due to the readily available data and the fact that QinetiQ's current models consider wind waves. Further expanding upon the impact of wind waves would allow for QinetiQ to expand upon their current systems.

This research project will identify the key approaches to understanding the effects of the sea surface on the performance of any sea surveillance radar. To achieve this ambition, existing wave models were explored, specifically WAVEWATCH III and Simulating WAves Nearshore (SWAN). These existing wave models are open source models that have the key equations for wave propagation inbuilt. These wave models are open source and offer a variety of predefined functions as well as the opportunity to input alternatives to reproduce a model close to the ground truth.

Additionally, an alternative approach was explored using two key algorithms for calculating the wave spectra, Pierson-Moskowitz and the Joint North Sea Wave Observation Project, JONSWAP. Wave spectra are the energy density of ocean waves, and is commonly used when calculating characteristics of any waves. Understanding the change in energy density and the variables responsible is a potential candidate for developing wave models.

This project seeks to develop a wave model for the purpose of radar assurance activities. The end result will be to develop an improved understanding of waves and their characteristics, and to find a method of replicating real world data that can be inputted into a radar model.

2 Methodology

2.1 Understanding the Approach

To test the performance of a radar using a model, one variable considered is the significant wave height, the average height of the top 33% of waves. Waves are important as they are a significant source of clutter for radar, so to separate an object of interest from the waves, it is important to be able to characterise the waves via their properties.

The reason why only the top 33% of waves are interesting is because waves that reach those heights have the largest effect on a radar, and the smaller waves are negligible [1]. There were two approaches to this goal, firstly to model the propagation of the waves, and secondly, to model the distribution of the waves. Being able to model the propagation of the waves would theoretically yield consistent, representative results, understanding what causes the formation of a wave and how the environment impacts it. On the other hand, modelling the distribution could be deemed a simpler approach that would provide similar results.

2.2 Modelling the Propagation of Wind Waves

Modelling the propagation of waves is the most promising approach. There are a variety of factors that cause the generation of waves, be that the transfer of energy from the wind to the sea surface, or the dissipation of previously formed waves. This approach is the most promising because it separates those factors into separate terms (section 2.2.3), allowing for more comprehensive descriptions of each term, before combining them to calculate the energy density of the sea surface.

2.2.1 Existing Wave Models

One of the main routes to modelling the propagation of waves is to use a pre-existing wave model, due to complex nature of modern source functions (section 2.2.3). Third generation wave models consider the action density of the waves, which considers the wave number and depth of the sea. The action density is the measured wave activity within a set area and is important when modelling the propagation since it's conserved whereas the energy density suffers from dissipation [2]. The wave number is the number of complete waves cycles over the course of its wavelength ($k = \frac{2\pi}{\lambda}$, where λ is the wavelength) [3].

Following research conducted by the US Army Corp of Engineers in the Pacific Ocean [4], WAVEWATCH III was selected to test its viability in coastal waters. Additionally, SWAN was selected as a specialist model for coastal waters. To utilise these models, data regarding the coastlines and the seabed was necessary, as was selecting key source functions to describe the propagation of the waves.

WAVEWATCH III is a well recognised wave model that is affiliated with the National Oceanic and Astmospheric Administration (NOAA), a US agency responsible for forecasting the weather and monitoring the oceans. SWAN is a wavemodel produced by DELFT University of Technology. Unlike WAVEWATCH III, it is limited to coastal waters and does not hold well if used to model a fully developed open ocean.

Installing both WAVEWATCH III and SWAN required installing additional software, specifically CMake, NMake, Ninja, Perl, Lahneey. CMake is the software that builds up C and C++ code, NMake is Microsoft's version of CMake, Ninja is a smaller build system that utilises CMake, Perl is a 'feature-rich programming language', and Lahneey is a Fortran-90 (F90) Fortran Compiler.

2.2.2 Coastlines & Bathymetry

To constrain the wave models to Cardigan Bay, it was essential to find a dataset that identified the shorelines. Initially, the Global Self-consistent, Hierarchical, High-resolution Geography Database (GSHHG) [5] was looked into, which was referenced in articles related to WAVEWATCH III. This database is an amalgamation of the World Vector Shorelines, Central Intelligence Agency's World Data Bank II and the Atlas of the Cryosphere, and produces a polygon display of the separation between land and ocean.

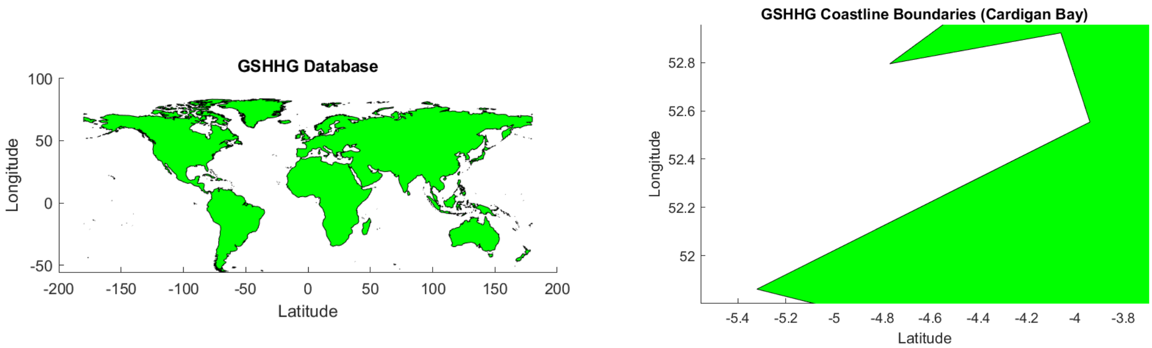

Figure 1: Plots of the GSHHG Database, specifically the L1 hierarchical level, which displays the boundaries between the land and ocean. The first plot is showing the entire L1 database, the second plot is constrained to Cardigan Bay. The x and y axis are latitude and longitude respectively, the green area represents land and the white area represents ocean. [5]

As seen by figure 1, this database is unrefined, and would most likely result in numerical errors. This warranted finding another source for shorelines. The solution was found in the General Bathymetric Chart of the Oceans (GEBCO) [6]. GEBCO not only offers information on the shorelines, but also information of depth of the water, essential for this project.

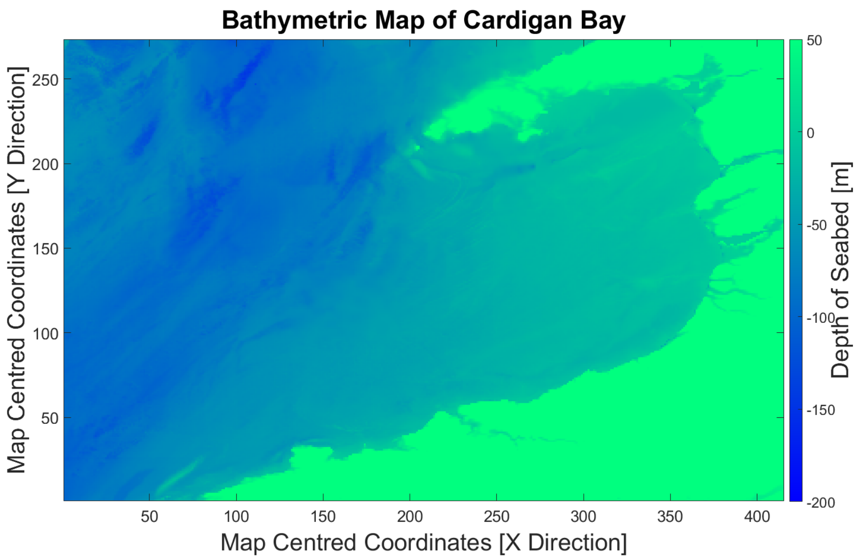

Figure 2: Bathymetric Map of Cardigan Bay. This displays an overview displays Cardigan Bay, with map centred x and y coordinates. The colour bar represents the height of the sea bed in relation to the mean sea level. [6]

2.2.3 Source Functions

Source functions are used to define the propagation of the waves. Whilst the fully developed open oceans tend to focus on the wind input (S_{in}), the nonlinear interactions (S_{nl}), and the dissipation due to white capping (S_{ds}), these functions are not directly relevant to coastal models. Whilst all wind waves are impacted by the wind input, the function has been designed for fully developed open oceans, and would require further testing. The dissipation due to white capping considers swell waves, waves that travel into the observation grid from a further distance, which are unlikely to be seen in Cardigan Bay due to the short distance that wind can travel along the sea surface for (fetch). Considering the behaviour of the waves, the entropy of seabed (as seen in figure 2), and the assumptions made for JONSWAP (see section 2.3.4); the wind input, the nonlinear interactions and the dissipation due to bottom friction have been deemed the key functions for Cardigan Bay.

$$S_{in}(k,\theta) = \frac{\rho_a}{\rho_w}\sigma(k)\gamma(k,\theta)F(k,\theta) \tag{1}$$

where k is the wave number, θ is the direction of propagation, ρ_a and ρ_w are the densities of the air and water, σ is the intrinsic frequency, γ is the temporal growth rate and F is the wave number spectrum [7].

$$S_{nl}(f,\theta) = \alpha^3 g^2 f_m^{-4} \hat{S}_{nl} \tag{2}$$

where f is the frequency, θ is the direction of propagation, g is the gravitational field strength, α and f_m are the energy and frequency scales, and $\hat{S}_{nl}$ is the non-linear distribution [8].

$$S_{bot} = -C_f \frac{\sigma^2}{g^2 \sinh^2 kd} E(\sigma,\theta) \tag{3}$$

where S_{bot} is the source function for the friction of the sea bed, C_f is the dissipation coefficient, σ is the radian frequency, g is the gravitational field strength, d is the depth of the seabed, E is the spectral density (energy), and θ is the direction of propagation [9].

These equations are incorporated into an equation for the energy density function,

$$F = S_{in} + S_{nl} + S_{bot} \tag{4}$$

where F is the energy density of the sea surface, S_{in} is the source function for wind input, S_{nl} is the source function for nonlinear interactions, and S_{bot} is the source function for the dissipation due to bottom friction [10]. At this point, alternative source functions such as dissipation due to white capping can be brought back into the model if required, by adding additional terms to the energy density equation.

As previously discussed, dissipation due to bottom friction is required for Cardigan Bay, as the seabed is shallow and is uneven; this would cause drag to the wave currents.

2.2.4 Creating Grids and Key Algorithms

Grids can be considered as smaller wave models being combined to produce the overall wave model. These grids also help alleviate the numerical errors resulted by the potential obstructions in the sea. These obstructions include, but are not limited to, small islands, ice and barrier reefs [11]. WAVEWATCH III uses rectangular grids of different resolutions and SWAN is capable of using both rectangular grids and triangular grids [12]. Higher resolution grids provide more details than lower resolution grids, however they are more computationally demanding.

The mosaic approach is the algorithm proposed by Tolman for combining numerous smaller, high resolution grids [13]. This is a key algorithm, particularly in both littoral environments and coastal waters where using high resolution grids can reduce the effect of obstructions and the shoreline. Two way nesting is an algorithm that supports the mosaic approach, as it allows data to be transferred from low resolution grids to higher resolution grids and vice versa [13].

Whilst Cardigan Bay has a limited amount of obstructions, grids, the mosaic approach and two way nesting are essential algorithms to consider. Should the project expand to other parts of the British coastlines, there is a potential that the wave model will have to consider many obstructions. Littoral environments such as Poole Harbour and regions of Scotland are littered with small islands, which would disrupt the flow of energy inside the waves. Both SWAN and WAVEWATCH III take different approaches to this. SWAN uses an algorithm called hybrid grids, which is the combination of rectangular and triangular grids that can shape themselves around these obstructions, and would allow for the model to get closer to these obstructions without losing details. WAVEWATCH III calculates the decrease in energy due to these obstructions with $S = 1 - \alpha$, where S is the fraction of spectral density and α is the transparency [11].

2.3 Modelling the Distribution of Energy

Whilst the source functions mentioned in section 2.2.3 do describe the distribution of energy, there is an alternative approach that fell within the scope of this project. This alternative approach is to evaluate two well recognised algorithms, Pierson-Moskowitz and the Joint North Sea Wave Observation Project (JONSWAP), both of which describe how the energy is distributed along the waves.

2.3.1 MATLAB Model

The aim was to develop a model of the sea surface, where the data can be input into a radar model. To validate the work, the graphs computed were compared to real world data. To build this model, a number of assumptions were made. Firstly, it was assumed that the height at which the wind speed was measured was equal at any given height above mean sea level. Secondly, it was assumed that the single data source (Waverider) was representative of the entire sea surface of Cardigan Bay. Finally, that all angular frequencies were examined instead of identifying the relevant angular frequency for a given data point (wind speed).

This project was computed in Matlab, because of the software's capability to work with large databases, and the plethora of in-built functions. A single script was used as the dataset was called in and used in multiple points across the script, as such to prevent potential confusion. Within the main script, functions were created to define the wave spectra from both the Pierson-Moskowitz and JONSWAP equations, as well as the relevant significant wave heights.

Most notably, Matlab's integration function, imagesc and surf functions have been foundational for this project. Whilst it is possible to code the integration by initially completing them by hand and then coding the results, Matlab's in-built integration function has saved time and effort. For the majority of plots, using Matlab's in-built imagesc and surf functions has provided high quality information and clear data.

When computing the GSHHG database, one toolbox necessary to use was the nctoolbox, since it provides access to different file formats. This is important because GSHHG uses American Standard Code (ASC) files and Network Common Data Form (NetCDF, .nc) files, and the only way to read these files into the code is with a toolbox.

2.3.2 Database

The range at Aberporth supplied this project with data from the 1st and 2nd June, which contained data gathered by a Waverider buoy and Rangehead (weather station) in Cardigan Bay. This data was gathered by the MetOffice, and has been previously used for radar assurance activities by QinetiQ. This was beneficial for testing the current models that QinetiQ uses and comparing the models researched in this project. As this allows a comparison between methods to assess if improvements are achievable. The source is reliable, as the data is owned and monitored by the UK Met Office. Permission was given by QinetiQ for this data to be used within this project.

The variables used in this research project are the mean wind speed and the significant wave height. It is important to note that the mean wind speeds were in knots, so the data had to be converted into metres per second, by multiplying the values by a conversion factor of 0.514444. Knots is a common nautical unit for sailors and other professions in fields surrounding the ocean to work with, however all the equations and algorithms used metres per second, so converting the data was required.

2.3.3 Pierson-Moskowitz

The Pierson-Moskowitz equation is considered the basic equation for calculating the spectral density of wind waves. It considers the frequency of the waves, the effect of gravity and the wind speed. The wind speed is used to calculate the peak angular frequency.

$$\omega_0 = \frac{g}{U_{19.5}} \tag{5}$$

where ω_0 is the peak angular frequency, g is the gravitational field strength and $U_{19.5}$ is the mean wind speed at 19.5m above the mean sea level.

Then, by using an array of angular frequencies, ranging between 0 and 2π, using the Pierson-Moskowitz equation will show the relation between angular frequency and the mean wind speed.

$$S(\omega) = \frac{\alpha g^2}{\omega^5} exp(-\beta(\frac{\omega_0}{\omega})^4) \tag{6}$$

where $S(\omega)$ is the spectral density, α and β are constants with values "$8.1x10^{-3}$" and "0.74" respectively, ω is the angular frequency and ω_0 is the peak angular frequency [14].

2.3.4 Joint North Sea Wave Observation Project, JONSWAP

JONSWAP was produced a decade later, with testing being conducted in the North Sea. This algorithm is of particular interest because the North Sea borders the United Kingdom. The average depth of the North Sea is approximately 90 metres [15]. There are a lot of similarities between the North Sea and Cardigan Bay, however the average significant wave height in the North Sea can vary between 0 and 11 metres depending on the part of the sea and the climate [16].

$$S_j(\omega) = \frac{\alpha g^2}{\omega^5} exp[-\frac{5}{4}(\frac{\omega_p}{\omega})^4]\gamma^r \tag{7}$$

where $S(\omega)$ is the spectral density, α is a constant, g is the gravitational field strength, ω is the angular frequency, ω_p is the peak angular frequency, γ is the peak enhancement factor with a value of 3.3 and has a factor of 'r', as defined in equation 8 [17].

The peak enhancement factor constrains the wave spectra and significant wave height, as it considers the self-stabilising nature of the waves [17].

$$r = exp[-\frac{(\omega - \omega_p)^2}{2\sigma^2\omega_p^2}] \tag{8}$$

where r is a factor, ω is the angular frequency, ω_p is the peak angular frequency, and σ is the width of the energy distribution around the spectral peak [17].

2.4 Significant Wave Height

Calculating the significant wave height is key for radar models. To do this, it is important to calculate the energy of the waves.

$$< \xi^2 >= \int_0^\infty S(\omega)\, d\omega \tag{9}$$

where ξ^2 is the energy, S is the spectral density from either equation 6 or equation 7, and ω is the angular frequency [4].

Then to calculate the significant wave height using equation 10,

$$H_s = 4 < \xi^2 >^{1/2} \tag{10}$$

where H_s is the significant wave height [4].

An additional method would be to use the wind speed to calculate the significant wave height, as described in Bretschneider (1951) [18]. These equations (11) offer a potential solution for finding a method that can replicate the results from the ground truth.

$$H_s = 0.21\frac{(U_{19.5})^2}{g} \tag{11}$$

where $U_{19.5}$ is the mean wind speed at 19.5m above the mean sea level, which was the height of the instruments onboard the ship of Pierson and Moskowitz's initial paper.

Considering equation 11 stems from the research conducted by Pierson and Moskowitz, and how modern wave models assume the wind speed is observed at 10m above mean sea level, equation 12 should yield closer results to the ground truth.

$$H_s \sim 0.22 \frac{(U_{10})^2}{g} \tag{12}$$

where U_{10} is the mean wind speed at 10m above the mean sea level, which is the conventional height at which the mean wind speed is used in most current models, for example WAVEWATCH III.

These equations are useful, though the rangehead used in the database records the mean wind speed at 440ft ($\sim 134m$) above the mean sea level, therefore it's important to consider the effect of drag and that the value of the mean wind speed at 440ft is different than at 10m above the mean sea level.

Then, using the wind profile power law,

$$u = u_r(\frac{z}{z_r})^{\alpha} \tag{13}$$

where u is the unknown wind speed at height z, u_r is the known wind speed at height z_r, and α is the atmospheric stability [19].

By substituting in the values of the wind speed at 440 ft above the mean sea level into equation 13, then using equation 5, the results should have a closer resemblance to the ground truth.

3 Results and Discussions

3.1 Introduction

This project aimed to model the sea surface of Cardigan Bay. Following on from the algorithms and theories discussed in chapter 2, this section will explore the opportunities, limitations, and the results from this research. By the end of this chapter, there will be an understanding of WAVEWATCH III and SWAN as two key wave models. There will be also be an understanding of the Pierson-Moskowitz and JONSWAP wave spectra equations and which is more representative of Cardigan Bay.

3.2 Modelling the Propagation of Wind Waves

The results from WAVEWATCH III and the SWAN wave models were unobtainable, due to technical issues. When implementing the software, it was discovered that Lahey is a legacy compiler and is no longer available. After attempting to implement both AMD's fortran compiler (AOCC) and the GFortran compiler (GNU Compiler Collection, GCC), there were still errors. Resolving this issue fell outside the scope of this project. However, both models showed promise, and more research was conducted surrounding both.

The documentation for WAVEWATCH III shows a variety of options that would allow for the model to be constrained to Cardigan Bay. One point of interest is how the options for grids have developed. Initial research indicated that WAVEWATCH III exclusively used rectangular grids, however due to the consistent development, both curvilinear and triangular grids are options [20].

After reviewing the documentation for both models, nesting SWAN within WAVEWATCH III would yield the greatest results. WAVEWATCH III was deemed the most promising for a fully developed open ocean, and SWAN was specifically produced for coastal waters by a specialist team. Additionally, SWAN's hybrid grids (combination of rectangular and triangular grids) would potentially limit the effects of obstructions far greater than WAVEWATCH III's algorithms would.

As an alternative way of exploring this approach, research was conducted into examples produced by experts in the field. One such example is based off of SWAN, and was produced by Dr Christopher Sherwood of the United States Geological Survey (USGS). This example outputted graphs depicting the significant wave height and wave period. Using this example, it was possible to see how to compute the grids and relevant functions necessary for a sophisticated wave model [21].

The model produced by Chris Sherwood depicted four characteristics of of an unknown bay in America [20]. For this project, the key characteristic was the significant wave height, where a 2D plot was displayed, with the x and y axis representing the latitude and longitude, and arrows that depicted the direction of wave propagation. One point of interest is the grids that are visible throughout this model. Because the grids are high resolution, the change in significant wave height is clearly observable.

Additionally, there was a website for a project called "Earth" that used WAVEWATCH III and had hindcast data [22]. This site only offered results for three hour time intervals, causing it to be a poor candidate for modelling the sea surface. When considering the purpose of this project, having three hour intervals would limit the time that the range can be operational, and considering additional factors that might limit range activity, three hour windows is not viable. Whilst linear interpolation between points was considered, due to the nature of the sea surface, it would not be viable to assume that the waves rise and fall as expected. This website was useful as it does demonstrate that models like WAVEWATCH III and SWAN are strong candidates and worth pursuing further, as this is only a feature of the website and not the model itself.

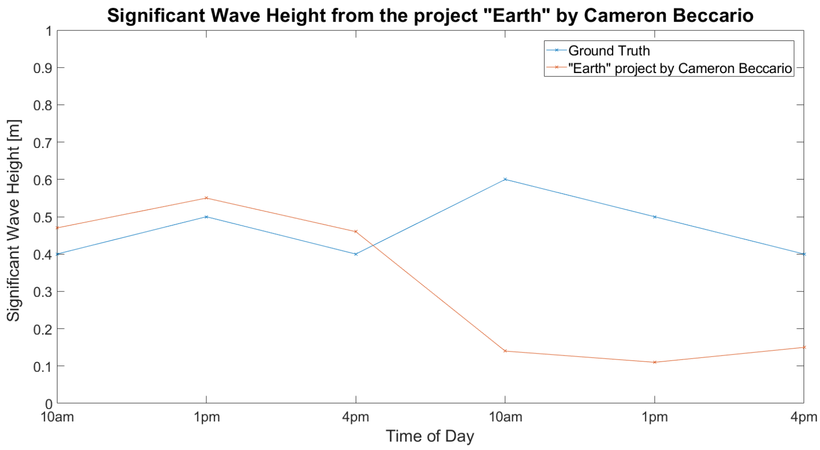

Figure 3: This plot displays the difference between the ground truth and the results supplied by the 'Earth' project.

Figure 3 displays the difference between the ground truth and the data received from. This solidifies the viability of using a pre-existing wave model. It is important to note that the exact location of the Waverider buoy is considered land on this website. This is to be expected because some wave models will consider very shallow water as land to avoid numerical errors, however there is a limitation where the wave heights in two different, yet very close, locations are different.

Overall, SWAN stood out the most, as a model its a specialises in coastal waters whereas WAVEWATCH III tends to be more generic.

3.3 Modelling the Distribution of Energy

Following the equations in section 2.3.3, section 2.3.4 and section 2.4, this approach modelled the significant wave height by calculating the spectral density. The spectral density was calculated by using the Pierson-Moskowitz and the JONSWAP equations. Compared to the previous approach, considering the energy in waves yielded promising results.

3.3.1 Pierson-Moskowitz

The results from the wave spectra equations were interesting, exhibiting behaviours expected from waves. The graphs display how the waves increase as the mean wind speed increases, which is to be expected because energy from the wind is being transferred to the waves. It was clear that additional factors were necessary to consider, for example the fetch.

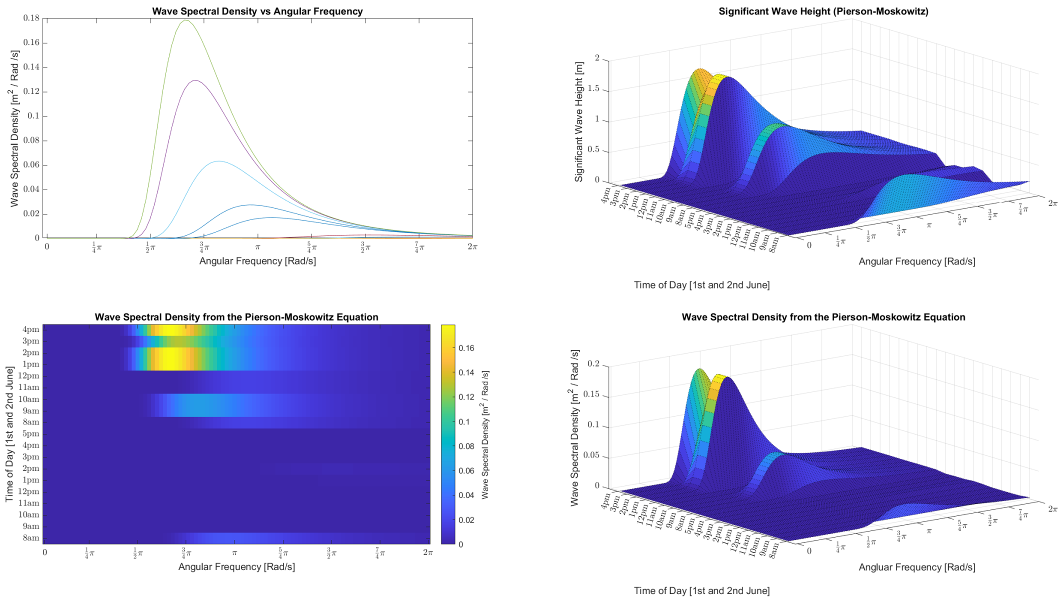

Figure 4: Plots showing the change in wave spectra and significant wave height whilst using the Pierson-Moskowitz equation. The plot in the top left displays the change in wave spectra in relation to angular frequency. Each line represents a mean wind speed, in relation to the data set. The graphs underneath display the wave spectra with respect to the angular frequency and the time of day. The colour bar on the graph on the left displays the value of the spectral density. The top right plot displays the significant wave height with respect to the angular frequency and time of day.

The graphs in figure 4 describe the relation between the angular frequency and the significant wave height. The graph on the top left depicts the spectral density with respect to the angular frequency. Each line represents a different mean wind speed, in accordance to the real world data. This graph is important for identifying the peak frequency of the waves.

The two graphs underneath depict the the wave spectra in relation to the angular frequency and time of day. These graphs were important for understanding how the energy density changed over time. The graph on the left is 2D to give an better view of change in angular frequency and where the peaks are. The colourbar indicates the value of the wave spectra. The graph on the right is 3D to give a better representation of the steepness of the growth in wave spectra.

The graph on the top right depicts the significant wave height in relation to the angular frequency and time of day. This is only depicted in 3D as there's no need to view the exact relation between angular frequency and the peaks, mathematically those details should be a repeat of the graph with wave spectra, and 3D visually gives a better representation of significant wave height.

If we assume that the equation is flawless, then an assumption would be that there are two potential frequencies that suit this model. One step would be to find those necessary frequencies. Whilst the provided database does not provide a frequency, using the average and peak wave period an approximation could be calculated.

3.3.2 Joint North Sea Wave Observation Project, JONSWAP

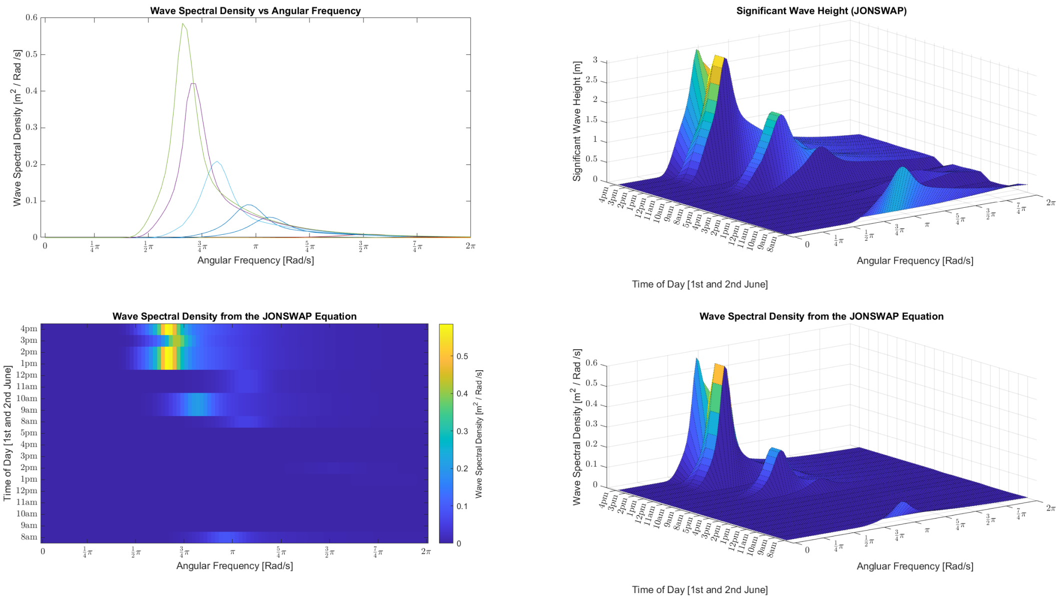

Figure 5: Plots showing the change in wave spectra and significant wave height whilst using the JONSWAP equation. The plot in the top left displays the change in wave spectra in relation to angular frequency. Each line represents a mean wind speed, in relation to the data set. The graphs underneath display the wave spectra with respect to the angular frequency and the time of day. The colour bar on the graph on the left displays the value of the spectral density. The top right plot displays the significant wave height with respect to the angular frequency and time of day.

Similarly to the graphs for the Pierson-Moskowitz method, the graphs in figure 5 describe the relation between the angular frequency and the significant wave height. The graph on the top left depicts the spectral density with respect to the angular frequency. Each of the lines represent a different mean wind speed, in accordance to the real world data. This graph is important for identifying the peak frequency of the waves.

The two graphs underneath depict the the wave spectra in relation to the angular frequency and time of day. These graphs were important for understanding how the energy density changed over time. The graph on the left is 2D to give an better view of change in angular frequency and position of the peaks. The colour bar indicates the value of the wave spectra. The graph on the right is 3D to give a better representation of the steepness of the growth in wave spectra.

The graph on the top right depicts the significant wave height in relation to the angular frequency and time of day. This is only depicted in 3D as there's no need to view the exact relation between angular frequency and the peaks, mathematically those details should be a repeat of the graph with wave spectra, and 3D visually gives a better representation of significant wave height.

3.3.3 Comparing Pierson-Moskowitz to JONSWAP

The key difference between the significant wave height from both the Pierson-Moskowitz and JONSWAP approaches is that JONSWAP's peaks are more constrained. This is due to the peak enhancement factor, and is a very useful term that describes the self stabilising behaviour of the waves. JONSWAP would benefit from including the fetch, which is the length of sea surface that wind can travel uninterrupted.

Attempts to include the fetch have been unsuccessful, as the wave spectra supposedly equals zero. This is not the case, however the code matches the equations. One potential source of error is that for testing, a single fetch was taken from the approximate location of waverider buoy to the Irish coastline, which spanned for 125km. To fully test the significance of the fetch, it would be important to create a database of distances from the waverider buoy to the nearest coastline for each potential direction of the wind.

3.3.4 Significant Wave Height

The results from the Pierson-Moskowitz and JONSWAP approaches gave a range of values. This is a due to a variety of frequencies being used, thus to compare both methods, the average and peak significant wave heights were compared to the hindcast data. This was to test which model would replicate the observed hindcast data consistently and would offer reliable predictions in the future.

The significant wave height was crucial due to the visual representation of the sea surface and it's widespread use in model validation. This visual representation was key in observing the waves behaviour, understanding the dependence on frequency, and identifying potential errors in future predictions.

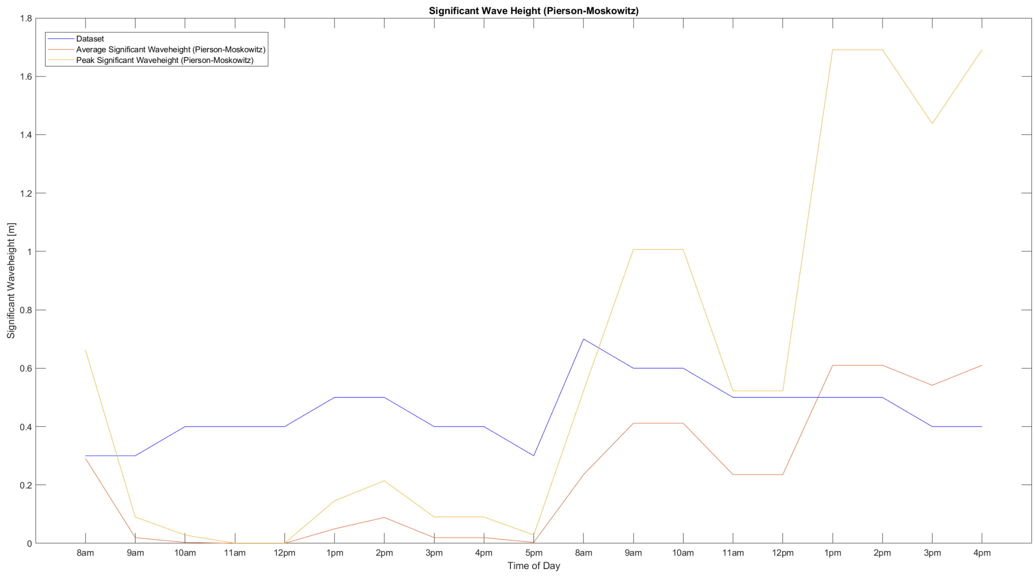

Figure 6: This graph displays the average and peak significant wave height in comparison to the ground truth. The significant wave height is on the y axis, and the time of day is on the x axis.

Figure 6 displays both the average and peak significant wave heights. For the 2nd June, the frequency lies between the average and peak frequencies, however the 1st June doesn't have a frequency that attains a wave height equivalent to that of the ground truth.

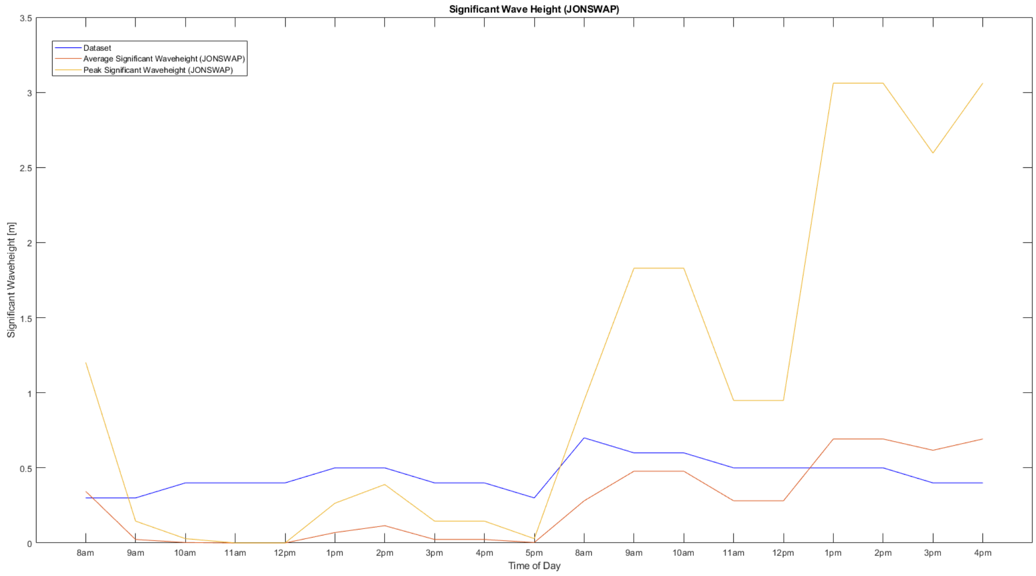

Figure 7: This graph displays the average and peak significant wave height in comparison to the ground truth. The significant wave height is on the y axis, and the time of day is on the x axis

Figure 7 shows that JONSWAP produces very similar results to the Pierson-Moskowitz spectra, however the peaks reach higher wave heights and they are more constrained. It is highly unlikely that Cardigan Bay would experience wave heights of $\sim 3.2m$. Following a document by Natural Resources Wales (NRW), Cardigan Bay experiences wave going over 1m approximately a quarter of time in June [23]. It is interesting that in the morning of the 2nd June, the ideal angular frequency would lie between the peak and average , but in the afternoon, the ideal angular frequency would be less than average.

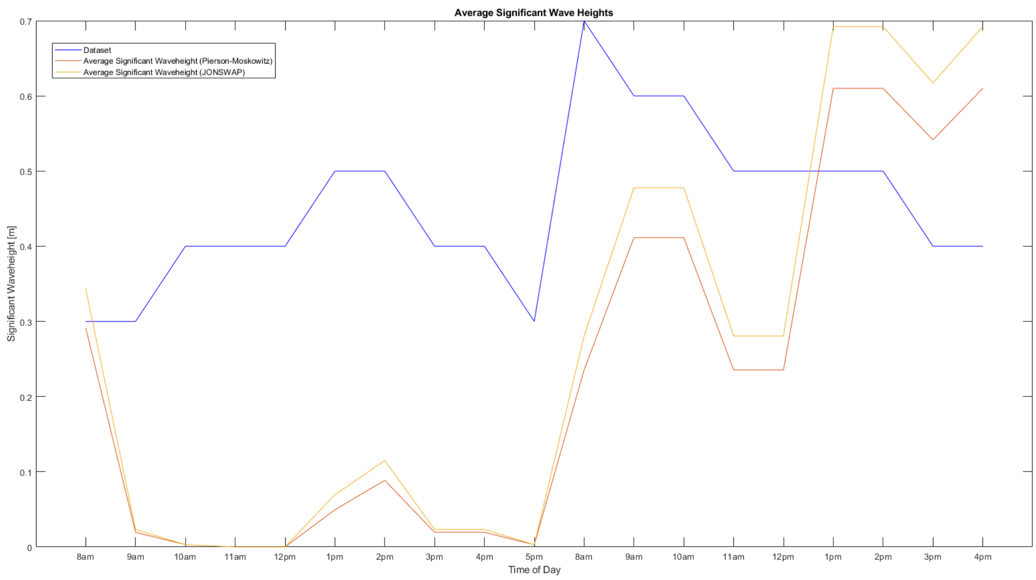

Figure 8: This graph displays the average significant wave heights from both the Pierson-Moskowitz and JONSWAP approaches.

From figure 8, it is clear on average JONSWAP is a more reliable than Pierson-Moskowitz. This is due to the consideration of the self-stabilising behaviour of waves, which can be seen by the closer representation of the average significant wave height [16]. A point of concern is that neither spectra follows the behaviour of the observed significant wave height. This could be due to the fact that neither the Pierson-Moskowitz nor the JONSWAP approach considers the friction caused by the seabed, or because the fetch hasn't been incorporated successfully.

Following equations 11, 12, and by correcting for wind speed with 13, it is clear that the wind speed is a key variable for measuring the significant wave height.

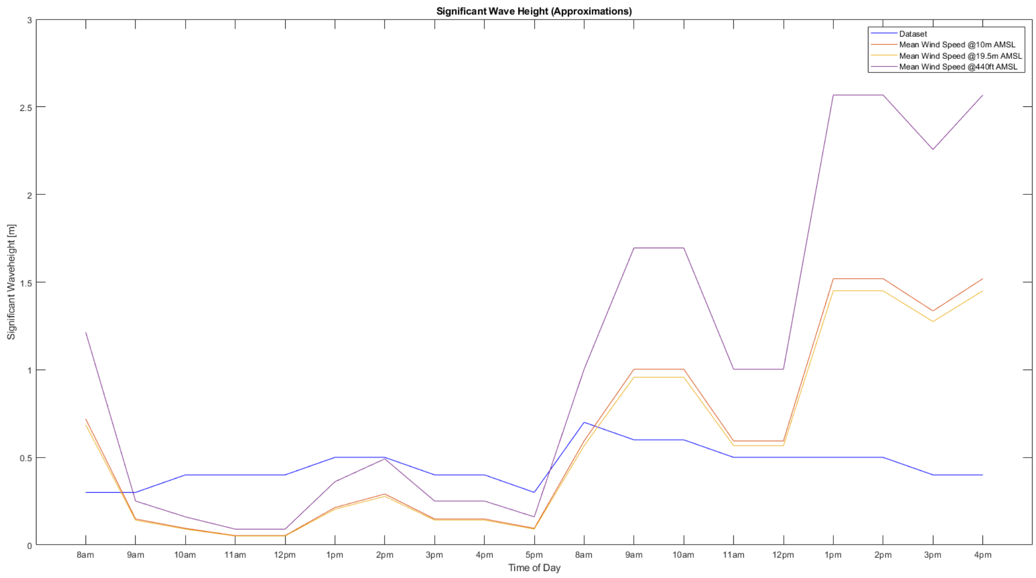

Figure 9: This graph displays the approximate significant wave height equations for different wave speed heights.

Figure 9 confirms the speculation that the height of the mean wind speed changes the results when applied to calculating the significant wave height. Excluding 8-9am, the 1st June had more representative results from the wind speed at 440ft, but the 2nd June had more representative results from the wind speed at 10m.

Overall, it is clear that the wind speed is a contributing factor to the significant wave height, however there are other factors to consider.

In all the approaches, there are points of concern. Points of concern lie between 8am and 9am on both the 1st June and 2nd June, then between both 12pm and 1pm, and between 3pm and 4pm on the 2nd June, where the change in mean wind speed does not match the behaviour of the significant wave height. There are plenty of possibilities that could lead to this. Firstly, the mean wind speed is not the only variable that affects the sea surface. Kitaigorodskii proposed in 1962 that the spectral density was affected by the frequency, gravity, the wind speed and the fetch [24]. In the approaches taken, the fetch hasn't been considered, which could be impactful due to the short distances that wind can travel.

4 Conclusions

4.1 Remarks on the state of research

The existing wave models showed promise, having detailed equations that describe the propagation of the waves. It was unfortunate that WAVEWATCH III and SWAN had technical issues. Both wave models could have had promising results. Installing these software have been chapters in articles by themselves, and thus fell outside the scope of this project. Fortunately, there were options for viewing the potential that these projects had, such as the 'Earth' project by Cameron Beccario and an example of SWAN from Dr Chris Sherwood. From my research, SWAN would better represent Cardigan Bay than WAVEWATCH III however if the model was to be expanded to the rest of the British coastline, there could potentially be numerical errors around waters like Beaufort's Dyke, a 300m trench in the waters between Britain and Ireland [25].

From the current data, if we assumed that the peak significant wave height represents the Cardigan Bay, then Pierson Moskowitz would be the correct approach. However, from figure 8, it can be assumed that the more representative model is JONSWAP. That would assume that the frequency of the waves would not necessarily be the peak frequency. It would also be safe to consider that the frequency changes, and so to find the ideal model would be to seek the frequency at each individual time, that could require using the average and peak wave period from the database.

4.2 Limitations

The first limitation is in using pre-existing wave models. Unfortunately, WAVEWATCH III had issues running the set up file, and SWAN (for the available operating system and CPU) required legacy software that is no longer available. To resolve the software issue, guidance was sought from Dr Marcel Zijlema, an associate professor working on SWAN. Dr Zijlema was asked for advice in installing SWAN and constraining the wave model to Cardigan Bay, as this felt to be the essential points to being able to use SWAN. Unfortunately no response was received.

Whilst some websites used WAVEWATCH III, they had their own limitations and would not have been viable. Fixing these limitations would have taken a long time and fell outside the scope of this project. However it was identified that the key issues were the start up commands for WAVEWATCH III and the F90 Compiler. Additionally, this meant that the parameters researched and identified, that would enable the replication of Cardigan Bay, could not be tested in this environment.

The database provided by Rangehead and Waverider buoy has some flaws that restricts the potential of the wave models. Firstly, the data is only from a singular point. Then considering the time intervals, data is given by the hour. Observing the difference between the mean wind speed and the max wind speed, so data points have max wind speeds 3x larger than the mean wind speed. By collecting data more frequency, it would improve on the replication of the ground truth and if there are discrepancies in the equations or code, allow them to be more easily identifiable.

4.3 Future Developments

Further research would lead to substantial results, i.e replicating the ground truth more closely than the current model QinetiQ uses. Firstly, being able to set up a pre-existing wave model, such as WAVEWATCH III and SWAN, by using a different operating system could produce the most effective results. Alternative operating systems have F90 compilers that are recognised and supported in the installation manuals of most wave models. Linux, in particular, was recommended for installing these wave models.

Having some prebuilt systems, such as WAVEWATCH III and SWAN, in place and being able to apply alternative equations makes the pre-existing wave models the most flexible approach. This would be more flexible because the sources of propagation of the waves would be tailored for any given sea surface.

Considering the second approach, further research into the JONSWAP algorithm, particularly into key factors such as the fetch and the height of the wind speed recorded would make significant improvements to the model produced in section 2.3.4. Including the fetch would require the construction of a database, as the distance from the Waverider buoy to the nearest coastline varies, depending on the direction of the wind. Similarly, with the wind speed, a database could be created using a weather balloon to record the wind speeds at various heights in the lower atmosphere. These are two key factors that haven't been implemented into the equations fully, and could potentially affect the results. This could lower the modelled significant wave height, due to the access to more accurate and refined data, the model would be improved, to account for the energy transferred from the wind to the wave potentially lowering the peak of the wave spectra and thus the significant wave height.

Further expanding from modelling the effects of the wind on the waves, it would be important to consider other factors such as the humidity, the temperature of the air, and the atmospheric pressure (specifically the mean sea level pressure). These variables are recorded by the Rangehead, however the Rangehead is not in Cardigan Bay, it is on the shore and there is only one data set currently available to the project. More data could be gathered to further validate and improve upon the models.

4.4 Recommendations

Following the results collected thus far, the most ideal approach is to nest SWAN inside of WAVEWATCH III. This would offer the opportunity to model the British coastline, which has depths of less than 100 metres, and to model open oceans. This approach is limited by the complex software and the terms of usage. Unfortunately, fully exploring this approach was not within the scope of this project.

The second best approach would be to use either the Pierson-Moskowitz or JONSWAP spectrum. Whilst the peak significant wave height was better represented by the Pierson-Moskowitz equation, the average significant wave height was better represented by JONSWAP, and JONSWAP has the relevant research to consider the fetch. This approach would be more accessible than utilising a pre-existing wave , but wouldn't have offered the same flexibility.

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Appendix

MatLab Code

For the full code, visit: https://github.com/BenChurchillUK/ModellingTheSeaSurfaceOfCardiganBay

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