

# Last mile delivery: optimization model for drone-enabled vehicle routing problem

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**Abstract** – In light of e-commerce's exponential growth trajectory over recent years, it has never been more critical to evaluate the last-mile delivery segment for its efficiency and cost-effectiveness. Autonomous vehicles (drones) offer considerable promise as potential solutions with ongoing investigations into emerging technologies in this context. To address problems related to last mile-delivery in logistics operations, the practicality of adopting a hybrid truck-drone delivery system is examined through this study. The researchers utilized Mixed-integer linear programming (MILP) and Gurobi optimization solvers both for optimizing performance as well as facilitating execution. While testing a drone dataset of a well-known logistics company as part of their research using an optimization model, the findings suggested that were remarkable competitive advantages including significant gains in reduction of timing. Nevertheless, there are several constraints like maintenance, recharging, difficult weather conditions & traffic congestion-necessitating focused innovative AI-based approaches. In spite of these impediments, a hybrid truck-drone's potential applicability can remarkably boost the efficiency of last-mile delivery operations.

**Keywords** – Last Mile Delivery; Vehicle Routing Problem, Truck-Drone Delivery

## 1.1 Introduction

The importance of last mile delivery cannot be emphasized enough in today's vast e-commerce market, projected to reach a staggering \$7.4 trillion by 2025. Over the past decade businesses have strived to find effective and cost-efficient solutions to overcome the challenges associated with last mile delivery. As competition intensifies in this industry companies offer lucrative options like same day delivery and 30-minute delivery to attract more customers. With advancements in robotic and AI technology, e-commerce businesses are putting trial and error methods into practice to optimize their operations. Incorporating innovative ways of delivering products is one of these methods with self-driving autonomous vehicles such as drones and delivery robots being of particular interest (Aurambout, Gkoumas & Ciuffo 2019). In fact, Amazon was the first corporation to propose the use of drones for its delivery services back in 2013 (CBS Interactive Inc., 2013). UPS obtained a full drone operating license from the Federal Aviation Administration (FAA) in October 2019 – making history as the first company worldwide to do so – allowing them to extend their existing small drone delivery service pilots into a nationwide network (UPS, 2019). In addition to industry giants like UPS and Amazon freshly established startups are also taking part in real time delivery using drones. Flirtey is among those startups who started operations back in 2013 as an urban drone delivery service provider. Furthermore, Matternet out of Zurich utilizes drones for emergency medical deliveries while Zipline delivers donated blood successfully via drone: an impressive use case within the medical sector!

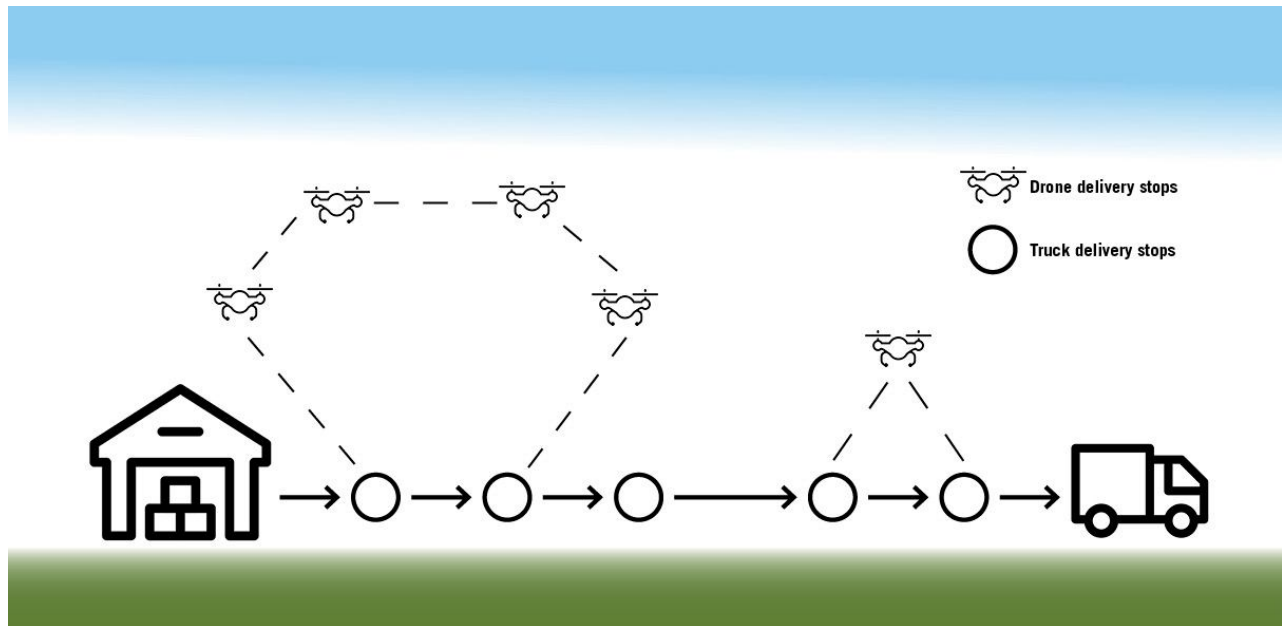


Figure 1.1: Truck-Drone Hybrid Delivery Source: (Floether, 2018)

When compared to trucks and vans, drones offer greater agility, are less impacted by traffic, and face fewer limitations from natural or human-made obstacles. Similarly, delivery robots designed for sidewalks enjoy the advantage of mobility without requiring parking space. These new technologies may appear to be ideal substitutes for traditional package transport methods (DHL, 2014). However, these innovative delivery options also have several limitations such as inclement weather conditions, power supply constraints, legal obligations, and cost-effectiveness. Therefore, manned vehicles such as trucks and vans maintain their relevance in package logistics. Owing to their size and historical status as a transportation mode approved by city planners, they offer a longer range of travel with no halt cost-effective transportation choice for commercial concerns. Additionally, ground-based vehicles are immune from weather-driven issues resulting in greater reliability.

### 1.2 Background of the study

The present study centers on investigating the strategic and practical overlaps existing between two modes of package delivery – truck delivery and drone delivery- with an aim towards exploring small-scale implementation possibilities. While trucks act as primary handlers, drones are used for secondary operations. The former transports packages right from distribution centers up to parking spaces where they await their final direct deliveries using drones. This report has a major focus on the subject of the Vehicle Routing Problem (VRP), which is an extensively researched topic in the field of operational research used for determining more efficient routes for fleets comprising secondary movers.

Unquestionably, the outcomes derived from this study will manifest consequences beyond its commercial applications. Grappling with life-saving logistics requirements by organizations involved in humanitarian aid, military operations and fire brigades who work with deadlines would benefit from our discoveries.

### 1.3 Objectives of the Project

As part of long-term commitment towards finding sustainable solutions for last mile deliveries, our aim is to work on a practical model of a truck-drone combined delivery system. By focusing on specific research areas, hoping to gain insights into how this technology can be further leveraged. The main objectives of the project:

Examining how combining both truck and drone technology could potentially optimize time management throughout delivery processes.

Also examining potential ramifications impacting societal well-being and commercial processes as a result.

Additionally, through this study we hope to identify opportunities where lessening carbon footprints could become possible through combined usage while also strengthening environmental security.

### 1.4 Research Questions

This study is undertaken to understand the following questions:

How can hybrid truck-drone delivery system solve the last-mile delivery problem?

How does the truck-drone combination help to reduce the cost of last-mile delivery?

How businesses and society can be benefited from truck-drone delivery?

How does a truck-drone delivery model reduce Co2 footprint?

### 1.5 Significance of the Study

In this paper, our focus area is "last mile delivery" - a crucial domain for both commercial and community entities alike. Our aim is to present feasible solutions that would yield mutual benefits on a global level. Across various industries, innovative technologies are transforming traditional practices and logistics is no exception. Among them are advanced drones that enable efficient delivery management and contribute significantly towards driving growth by overcoming existing challenges effectively. Drone reliability within logistics has been extensively researched offering new perspectives for modernized operational processes worldwide by meeting performance expectations consistently. Efficiently delivering timely goods with minimal disruption hinges upon adopting measures such as site-to-site shipment facilitated through drones in ways promising substantive benefits including reducing anxiety among customers through timely delivery compared to conventional transportation options taking longer periods causing uncertainty versus faster modes implemented with proper planning & location-based capabilities ensuring better order control instead catering efficiently according needs of the markets demand cycles better suited to succeed in today's competitive global marketplace.

For researchers interested in deepening their knowledge on truck-drone hybrid delivery systems, we believe this report will provide valuable insights into the subject.

### 1.6 Limitations of the Study

This study was done as a part of an academic project so there is some limitation to this study. Although this study can be taken to the next level for further research. The main limitation of this study was time constraint. As an academic project, this thesis was time bound so we could experiment with drone operation in the real field to assess our model. Another limitation was the lack of opportunity to work closely with actual logistic companies, so this model is developed through previous data, assumptions, and hypothetical environments.

## 2. Literature Review

### 2.1 Introduction

This research takes a closer look at the intricate challenges associated with managing a hybrid truck-drone vehicle routing system stemming from capacitive vehicle routing problems. To maximize customer satisfaction levels while utilizing both trucks and drones for efficient delivery services simultaneously is no easy feat and requires a comprehensive understanding of various factors influencing drone performance along pre-defined routes regardless of if they were being managed by trucks or drones alone. Henceforth combining route management for these two modes makes this study distinct when compared with routine literature on typical vehicle routing methods. Random examples from real-world scenarios were used in this study and produced precise calculations of outcomes by the algorithm used.

### 2.2 Relevant Problems

In this section we explore the complexities surrounding the Vehicle Routing Problem (VRP). The challenges vary significantly depending on which type of problem we consider; examples include Travelling Salesman Problems, Stochastic Vehicle Routing Problems or ones involving Time Windows. Each presents a distinct set of issues related to optimising routes effectively. Examining these tasks more closely underlines the importance of carefully tailored solutions that are both efficient and effective catering appropriately to specific requirements for each problem version.

**2.2.1 Travelling Salesman Problem (TSP):** The Travelling Salesman Problem (TSP) has gained extensive attention as an optimization challenge over time. It entails finding the quickest route that reaches multiple cities before ending at the initial location. The TSP has been employed in various fields such as logistics planning, computer wiring and DNA sequencing among others. However, while it offers practical solutions in these areas, the basic model does not make allowances for factors like capacity limitations or using multiple trucks in transportation. NP-Hard problem

classification marks TPS's difficulty level when dealing with a vast number of towns; making computation efforts daunting (Applegate et al., 2006).

**2.2.2 Capacitated Vehicle Routing Problem (CVRP):** Transportation logistics managers must consider different factors when solving routing problems for delivery vehicles. In the case of the Capacitated Vehicle Routing Problem (CVRP) several vehicles with unique qualities must meet customer needs while not exceeding capacity. This adds complexity to the process of scheduling, managing, and purchasing multiple trucks – a problem that has been studied by Toth and Vigo, 2014 in many real-world industries such as garbage disposal, school bus routing, and delivery services. Careful planning is required to optimise individual vehicle performance within these transportation schemes that must operate under changing parameters that can test even seasoned drivers.

**2.2.3 Multiple Depot Vehicle Routing Problem (MDVRP):** When dealing with logistic systems involving many storage facilities, there may exist substantial efficiency challenges that need addressing. However, one novel solution is using Multi-Depot Vehicle Routing Problem (MDVRP), which goes beyond conventional CVRP techniques by affording more freedom of movement for vehicles.

Compared to standard approaches, using MDVRP has impressive advantages but is still challenging when ensuring accurate delivery processes between customer selection and vehicle operation across each warehouse location while maintaining delivery targets requirements for each site intact. Nevertheless, this challenge highlights the vital role played by MDVRP in enhancing productivity in large-scale logistics, according to Cordeau et al.'s research from 2007.

**2.2.4 Periodic Vehicle Routing Problem (PVRP):** Managing visits for clients requiring personalized attention within an extended period poses significant challenges without considering their individual needs when scheduling appointments. Fortunately, PVRP extends VRP by taking into account a multi-day planning horizon suitable for industries such as maintenance services or garbage collection where daily requirements differ based on customer preferences. However, incorporating both frequency and time duration further enhances complexity while solving this problem (Francis et al., 2008).

**2.2.5 Stochastic Vehicle Routing Problem (SVRP):** The sheer unpredictability caused by random variables such as travel times, customer demands, and service durations makes it difficult to solve the Vehicle Routing Problem (VRP). However, adding Stochastic Vehicle Routing Problem (SVRP) further compounds the challenge; designing dependable routes that can navigate this uncertainty becomes pivotal. The use of SVRP assumes even more importance when constant traffic congestion exists or there are sudden shifts in demand or variable periods of service time. Moreover, Gendreau et al. (1996) have found that dealing with stochastic elements intensifies challenges involved while improving problem-solving practicality.

**2.2.6 Vehicle Routing Problem with Time Windows (VRPTW):** In the domain of Vehicle Routing Problem with Time Windows (VRPTW), it is necessary to ensure that every individual customer receives a satisfactory service within the respective time slot. Such a constraint helps us to assure that our customers receive their required support at a suitable time. The primary objective is to strictly adhere to these predefined time windows while optimizing the total distance covered during the route. This problem has found crucial importance in industries like food delivery and courier services where on-time delivery plays a vital role. As cited by Solomon (1987), incorporating timely windows into route optimization algorithms significantly increases its complexity level.

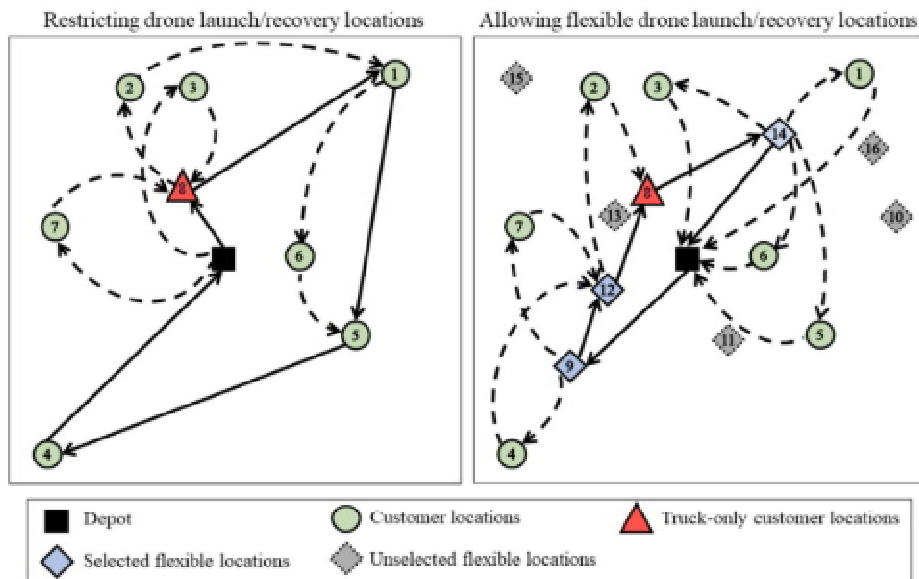
### 2.3 Empirical study

The application of drone technology in supply chains and logistics for last mile deliveries has gained significant attention through empirical research studies. To evaluate its usefulness' mathematical models were created using real-world scenarios while researchers conducted surveys assessing potential benefits associated with delivery systems supported by drones.

Innovative solutions put forth by Mohammad Moshref-Javadi et al., (2019) involves introducing trucks and drones into last-mile delivery systems a challenging undertaking due coordination difficulty between trucks and unmanned aerial vehicles (UAVs) involved within a singular system. However difficult it may be, the present solution provides potentially transformative change altering existing supply chain processes. Trucks would essentially become mobile depots capable of simultaneously launching and landing one or more UAVs for package deliveries while driving

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along their designated routes. The strategy is especially helpful in optimizing the expensive and complex last-mile delivery phase of the supply chain. Findings suggest that proper implementation of this approach has significant cost-saving potential—as high as 41%—yet poses challenges such as vehicle conversion, route planning, and varying Vehicle Routing Problem (VRP) structures.



**Figure 2.1:** Flexible location for Drone launching. (Source: www.sciencedirect.com)

Transportation systems today pose various challenges that require efficient mathematical models capable of handling hybrid vehicle-drone routing problems (HVDRP) to deliver commodities consistently and cost-effectively across major transport networks. Thus, stressing its importance unto us is Aline Karak and Khaled Abdelghany's 2019 study. Their method offers classification to clients based on specific delivery criteria that minimizes the expenses of deploying ground vehicles and unmanned aerial vehicles (UAVs). Their HYDRO framework can also generalize common vehicle routing problems while providing insights into customer trips across multiple depots, including HVDRPs that share similarities with ordinary VRP components.

Felix Tamke and Udo Buscher conversed on using drones in delivering items through conventional transportation networks, acknowledging that UAVs can speed up delivery times but highlighting the limitations imposed by constrained payloads and flight ranges. Nevertheless, their creative strategy of quick airborne transit, overcoming the challenges posed by HVDRPs while enhancing distribution system effectiveness. They further suggested a preprocessing method to determine the best flying speed for energy use as a potential solution to routing problems.

Mohammed Moshref-Javadi's initial idea was further developed by Mohamed R. Salama and Sharan Srinivas through an essential evaluation study in 2021 looking at drone operation rate generation while simultaneously addressing real-world issues faced during operations such as technical downtime. They provided insightful ideas on using truck-drone hybrid systems for last-mile package delivery in 2022 while providing realistic illustrations of how this technology might operate in real-world scenarios with governance constraints.

Cutting-edge technological solutions aimed at delivering superior consistency and efficiency for customers is one reason why large corporations like Google, DHL, FedEx, and Amazon are investing heavily in researching potential applications for their final deliveries.

It is imperative to have effective regulations in place that ensure strict adherence to technical standards. This emphasizes the importance of widespread usage or market acceptance, as rightly stressed by Salama and Srinivas. In terms of routing challenges, Shanshan Meng has shared fascinating insights. Her interpretation suggests various techniques that could lead to seamless pairings between trucks and drones, thereby enhancing process efficiency and delivering a powerful new experience that has the potential to transform the market. The use of both trucks and drones in this streamlined process has increased delivery radius and made the procedure smoother. However, one of the biggest hurdles in this area is ensuring regular supply times, which requires adequate maintenance and operational changes for energy optimization.

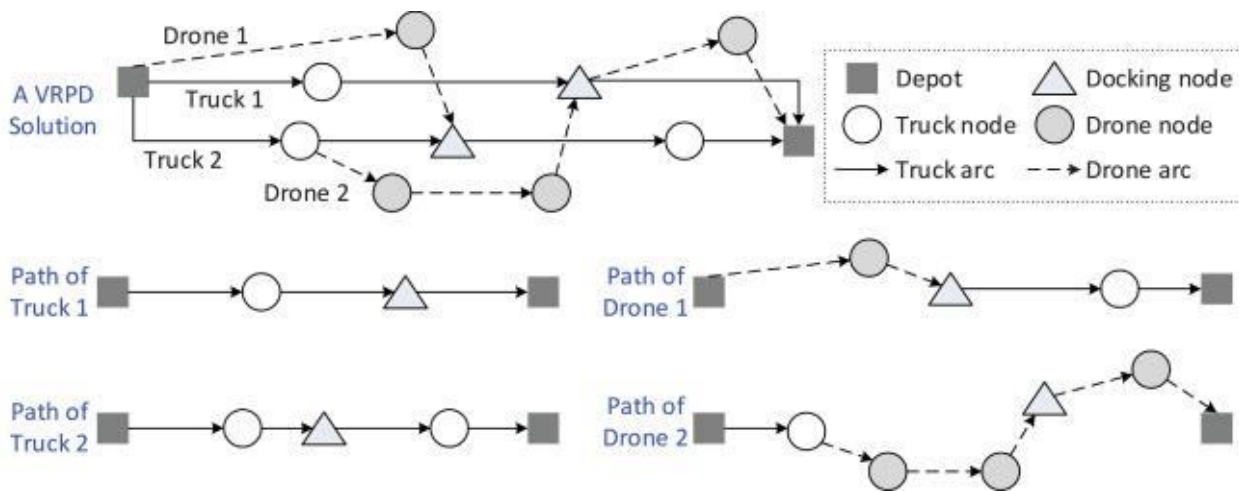


Figure 2.2: Routing problem with drones (Source: www.sciencedirect.com)

In light of increasing demands for drone delivery services, it's become imperative to consider one primary issue -the large amounts of energy required to operate these devices effectively. Indeed, the market demand for delivery services determines the minimum cost level economically viable to sustainably run drone-operated businesses while "customer set" includes all clients who drones could serve based on their scope with regards both accessibility and range.

Scholars from both academic institutions and corporation sectors remain curious with respect to how drone technology could potentially impact last-mile delivery systems positively; however, there are undoubtedly fundamental aspects worth considering before implementing such technology more widely. Unlike traditional vehicles, drones have several advantages, including faster traveling speeds when compared with slower modes of transportations such as conventional road vehicles being able operate autonomously lending itself particularly well when time-critical deliveries arise or take place in remote locations.

However, there are apparent limitations such as limited cargo capacity along with range contextualized by constraints associated mostly due to size or power compromise inherent within current-generation drone models. To ensure efficient use of resources when developing delivery routes planners must consider those limitations carefully along with the urban logistics model that has historically governed such planning processes. Drones have great potential as a viable last mile option in urban regions given that they offer quicker delivery at an affordable cost.

Despite its many benefits there is still significant red tape surrounding drone utilization because most drones rely on batteries for electricity. This dependency reduces both carrying capacity and operational range while increasing the need to develop new technologies capable of expanding battery life and cargo capabilities.

Dual vehicle systems combining drones and trucks present a promising solution for resolving vehicle routing problems but pose additional challenges associated with launch locations recovery procedures once airborne. Deploying drones from mobile truck bases could be an exciting way forward in developing efficient transportation and delivery infrastructure. As is the case with all different vehicles deployed for various logistical reasons, optimal routing approaches must be devised when considering drone mobility characteristics, i.e., operational range or payload capacity despite not being given sufficient attention in traditional vehicle routing methods. In putting together responsive strategies using overhead technology capabilities - we can optimize cost-savings and customer-centric efficiency for "last mile" deliveries by leveraging the perks of our favourite little flying helpers. Declaring ownership over challenges inherent to drone deployment can let us create custom solutions that work with their unique strengths while minimizing their limitations- making affordable yet high-performing delivery systems a scalable possibility we move ever forward into new tech iterations of industry 4.0.

## 2.4 Solution Methods

To solve the Vehicle Routing Problem (VRP), several approaches exist to provide a comprehensive overview of logistics and transportation processes. Four categories usually subsume VRP solutions: exact responses, heuristic

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answers, metaheuristic alternatives or hybrid methods. The choice of approach depends on variables such as complexity level availability of resources and desired accuracy rate in delivering solutions. These facts demonstrate that it takes diverse methodologies to overcome complicated logistical challenges. Numerous strategies are prevalent in handling VRP either individually or simultaneously while demonstrating their capabilities.

### 2.4.1 Exact Method

To efficiently solve a problem or issue at hand requires a systematic approach- one which eliminates potential solutions until only the best remains. In particular reference to the Vehicle Routing Problem (VRP), precise methods such as exact algorithms are used for this very purpose; where each and every possible route combination is analyzed in order to determine what is most efficient in terms of time spent and money earned or saved.

For example, techniques like branch-and-bound method utilize bounding algorithms to segment larger problems into easily manageable subproblems hence eliminating alternative options (Laporte,1992). On the other hand, dynamic programming converts complicated challenges into smaller and more concise ones addressed once over course program execution- ensuring that all previous solutions inform the collective outcome (Bellman1957). While Integer Programming conceptualizes VRP as system of decision-making integers working collaboratively with challenging optimization strategies taking any constraints presented by the situation under consideration. Looking at the downsides, when used on tough problems, this approach can be significantly inefficient and even lead to processing time wastage as research done by Wolsey in 1998 demonstrates.

### 2.4.2 Heuristic Method

In pursuit of swift yet practicable solutions to address complex obstacles such as vehicle routing problems, heuristics offer convenient rule-of-thumb procedures that are not necessarily ideal but strike a workable middle ground. Specifically contrived for Vehicle Routing Problems (VRPs), heuristics facilitate speedy identification of productive pathways. An example of how useful these techniques can be is found in Nearest Neighbor Heuristic which targets unvisited locations geographically closest to the designated depot while promptly contacting every customer on route. Insertion strategies start with single-route serving solo customer needs before gradually adding subsequent requests while evaluating predesigned cost-saving parameters tailored toward optimal savings via merged routes that result in comprehensive multi-route systems. Yet despite their nimble and flexible nature, it's important to note that the outcomes may not always balance out perfectly (Gendreau et al., 2002). A contrasting approach dubbed the Two-Phase-Technique builds initial solutions in one phase and then improves upon them in a second (Osman, 1993). While technically diverse and multi-dimensional, these methods still require significant computational resources.

### 2.4.3 Metaheuristic Method

Metaheuristics represent a more advanced method of search. They enable heuristics to avoid being trapped in local solution optima and widely investigate the solution space. One of these approaches is simulated annealing, which copies the lengthy cooling process utilized in metalworking (Kirkpatrick et al., 1983). By tolerating suboptimal solutions, this technique provides an escape route from local maxima. Genetic algorithms replicate natural evolution by retaining a population of possible solutions and developing new ones through genetic operators (Holland, 1975). Tabu search is a memory-based framework that avoids returning to previously reviewed solutions (Glover, 1986). Ant colony optimization imitates ant foraging activity by using pheromone trails to guide the search for food (Dorigo & Stutzle, 2004). Particle swarm optimization is another population-centered metaheuristic algorithm that replicates animal group behavior such as fish schools or bird flocks (Kennedy & Eberhart, 1995). Finally, bee algorithms are inspired by the way honeybee swarms search for food (Pham et al., 2005).

### 2.4.4 Hybrid Solution Method

Hybrid solutions combine different methods to maximize benefits and minimize shortcomings. For example, incorporating both heuristics and accurate procedures can produce excellent results in a shorter duration. Employing a hybrid approach for problem-solving in VRP is the truck-drone system, which uses trucks as carriers for numerous drones tasked with delivering goods. According to Murray and Chu (2015), this tactic combines drones' nimbleness and velocity with trucks' high capacity to optimize delivery efficiency. Moreover, another hybrid technique that merges machine learning with metaheuristics enhances problem-solving efficacy by predicting promising areas in solution space using machine learning models before analyzing it with metaheuristic algorithms (Liang & Smith, 2004).

## 2.5 Theories and Models

The use of drone-driven heuristic offers a fresh perspective on reducing the expenses linked with drone routing. By considering particular algorithms and defining practical actions, this method ensures utmost precision when creating drone routes. Furthermore, it guarantees accuracy in sending out drones and providing information from source stations through vehicles.

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### H6: The Drone-Driven Heuristic

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**Input:** Network topology and customers information

**Result:**  $S_V$  and  $S_D$

Set the closest station for each customer considering the stations in  $N_V$

$\psi_D$  = Calculate pair saving for customers

$\psi_V$  = Calculate pair saving for stations

$S_D$  = Build\_Drone\_Routes ( $\psi_D, w, r$ )

$S_V$  = Construct initial vehicle routes

**while**  $\psi_V \neq \emptyset$  **do**

    Starting from the first element,  $\vartheta_{ij}$ , of  $\psi_V$

    Get *route1* that contains station *i*, and *route2* that contains station *j* from  $S_V$

**if** (*route1*  $\neq$  *route2* & stations *i* and *j* are not intermediate nodes) **then**

*merged\_vehicle\_route* = Merge station *i* and station *j* in a new route

**if** (*merged\_vehicle\_route* is feasible from the drones' perspective) **then**

            Remove *route1* and *route2* from  $S_V$

            Add *merged\_vehicle\_route* to  $S_V$

**end if**

**end if**

    Eliminate  $\vartheta_{ij}$  from  $\psi_V$

**End**

**return**  $S_V$  and  $S_D$

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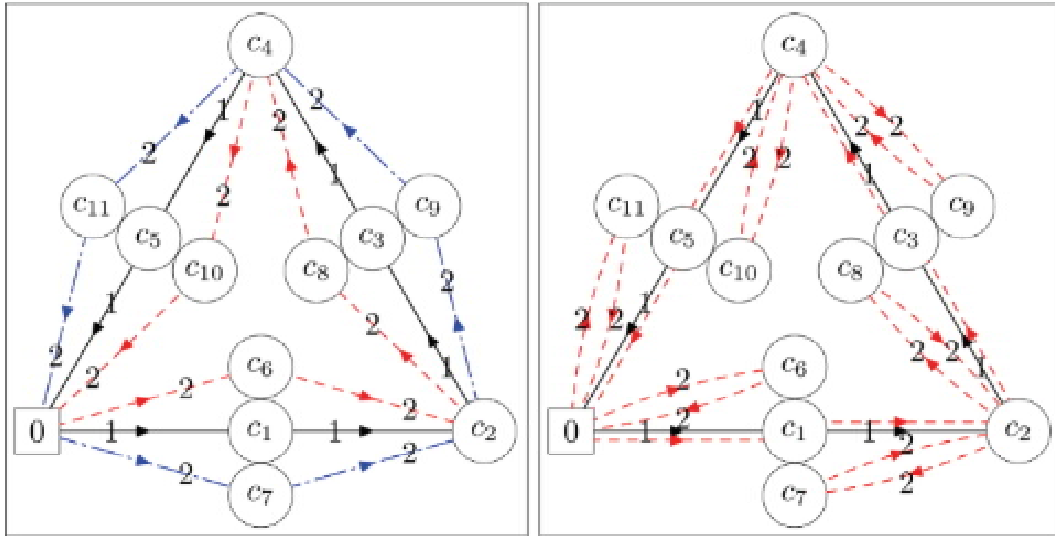
**Figure 2.3:** Steps adopted by the drone-driven approach. (Source: [www.elsevier.com/locate/trc](http://www.elsevier.com/locate/trc))

The usage of algorithm serves the purpose of calculating costs for multiple drone routes, resulting in a cost-saving matrix via drones. Supplementary data is derived from prior drone routes that are combined with recent data on drone routes, inclusive of information on pickup and drop-off stations for the drones (Redi et al.2020). To select a target node for a drone, the greedy algorithmic technique, accompanied by iterated local search using 2-opt, is most commonly utilized (Moadab et al.2022). In relation to time dependency in logistics networks, several academics have employed the iterative local search technique to find solutions. The intra-pair and inter-pair local searches are categorized based on how operators' function; the former aims to determine optimal travel paths for paired vehicles and drones.

Meanwhile, 2-opt algorithms aim to improve TDAP-TD RTT results by eliminating differences in cooperative routes. The widely known VRPD model assumes that truck fleets remain constant and there are fixed numbers of drones per track while attempting to locate paths requiring minimal time (Popovi et al., 2019). Drones landed at their take-off tracks while batteries are immediately charged after each landing according to Popovi et al., adding that many drones deployed simultaneously quickly tend to serve more consumers together.

Attempting to persuade an individual whose convictions supersede any other perspectives can prove unproductive. Communication with such people is often hindered by a lack of openness towards alternative opinions and evidence counteracting their beliefs. By validating dissimilarities respectfully and seeking agreement instead of perpetuating discord, conversations might move towards resolution more efficiently.





**Figure 2.4:** (a) Two slow drones of speed 2 (b) One faster drone of speed 4  
(Source: <https://onlinelibrary.wiley.com/cms/asset/>)

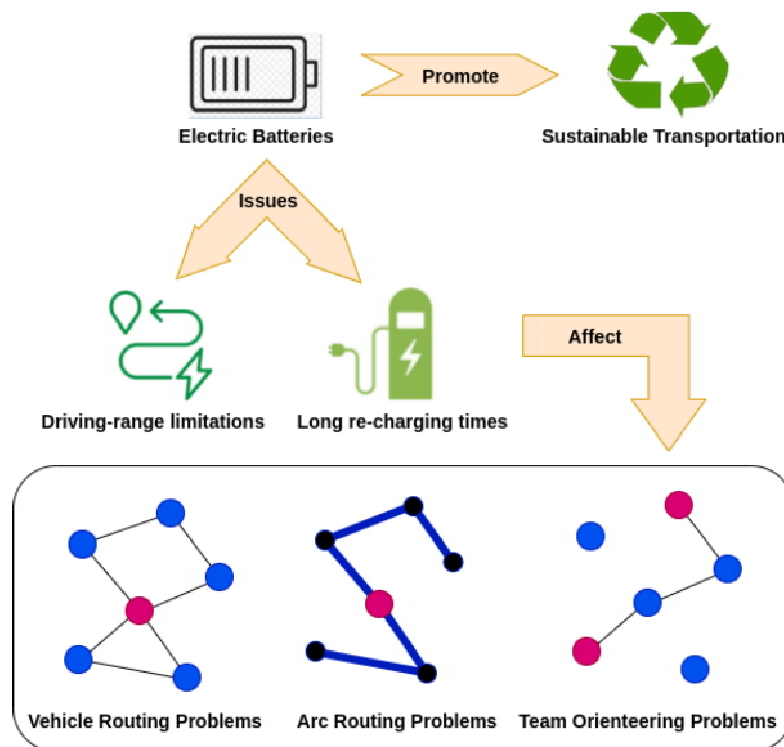
Another improved approach is by improvising the hybrid truck drone operations where LARO drone is allowed to follow along the track with the implementation of the greedy altering TSP - D routing by later and early obtaining of drones on the same space.

## 2.6 Literature gap

Retail companies have had trouble delivering things more quickly without significantly increasing the cost of transporting them. The use of drones in conjunction with more conventional approaches carried out by truck is one possible answer that has just come to light. Despite the fact that it is still in its infant stage, the hybrid technique shows promise for future investigation.

Liu and Y (2019) have out research that is considered to be ground-breaking in the field of vehicle-drone operations for last-mile deliveries. They proposed a model that is dependent on autonomous units that are charged with creating car-based docking stations in order to maximise the utilisation of logistical resources. However, in order to make the most of the potential benefits of this technology, it is necessary to resolve a number of important concerns first. Implementing well-organized processes while also doing exploratory research into cutting-edge technology is required in order to prevent drones from being overloaded while they are in flight.

In order to successfully integrate drones with the resources used for urban logistics, navigational concepts and strategic route planning need to be carefully studied. The existing body of research has not given sufficient consideration to the coordination of no-fly zones as a significant problem separate and apart from the privacy or security breaches related with drone delivery services. putting into action actual steps while considering both ethical and legal factors is absolutely necessary in order to protect individual rights against invasions of privacy or threats to security. As the number of customers who choose drone-based delivery services continues to rise, it is vital that any potential worries be proactively identified and addressed. There has been a significant amount of study conducted on the technical elements of drone delivery, such as noise pollution and public acceptability, amongst other operational issues; however, there has been little research conducted on the ethical implications of drone delivery there has not been a significant amount of emphasis dedicated to the examination of the effect of merging conventional distribution methods with drone technology. We will be able to develop synergy that works to everyone's advantage if we investigate potential new methods by which trucks and drones may work together to improve productivity while minimizing adverse effects on the environment. Previous research has shed significant light on the ways in which the integration of drones with conventional distribution methods may result in increased efficiency.



**Figure 2.5:** Electric vehicle routing by suitable transportation  
(Source: <https://www.mdpi.com/1996-1073/14/16/5131>)

However, further study is required in order to provide an accurate assessment of the benefits and drawbacks of using this strategy. For industry experts and researchers seeking for novel solutions in this field, the implementation of efficient drone delivery systems that are also socially acceptable is a good place to start and will play a very significant role.

## 2.7 Recommendation or implementation

Utilizing k MDVRP is an integral part of simplifying and managing restrictions outlined in the literature. This is particularly important for drones with a "multiple drain function," where any parameter does not decrease with each location pair. An alteration in the heuristic algorithm opens up new opportunities for future research (Zeng et al., 2022). It's important to note that the success factor used in Euclidean TSP may differ from that used while visiting clients with drones. Additionally, a suboptimal block partitioning strategy is employed to allocate drones based on client orders. The speed of the drone can also be adjusted using the RTS heuristic architecture.

The unmanned aerial vehicle (UAV) is an essential component utilized across various business sectors and its assembly involves assembling laser technology that ensures reliability and high value (Bakir et al., 2020). Identifying potential hazards through drone monitoring systems aids in preventing hazardous situations (Jeong et al., 2019). The use of portal monitoring systems and safety standards within their respective regions are imperative to ensure safety. Currently work on automation procedures for drones is ongoing with a potential emphasis on reducing errors (Bakir et al., 2020). The Dropped method offers a notable advantage as it expedites the inspection process and enhances the quality of findings. This is highly beneficial for ongoing studies identifying the optimal model. Additionally, this innovation simplifies the delivery procedure, which was previously more complex. According to Farajzadeh et al. (2020), this approach helps identify issues and determine appropriate solutions. The VRP has two key components: branch method and cut technique. Failure to implement varied techniques may result in reduced efficacy. One well-established solution to VRP is the "Clark and Wright savings algorithm," which considers all elements of vehicle routing, resulting in more efficient system functioning.

## 2.8 Summary

As critical as it may seem, the velocity at which a drone carries out its task of package delivery hinges on a single

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sheet of paper. This crucial document is produced via an online platform that provides diverse download speeds for different components to suit user preferences. These challenges have been linked to hitches experienced with primary drones and problems with routing systems and speed selection. The previous stages, which accentuated intensified work rate and delivery tweaking, culminated in this latest phase requiring enhanced energy input due to heightened drone speed. Through the adoption of Mixed-Integer Linear Programming (MILP) system, logistic teams can now adjust drone speeds while eliminating variables. This ensures optimal balance between economic gains and environmental impact regulations.

3. Methodology

3.1 Problem Description

To ensure accurate and timely parcel distribution in an area with multiple demand nodes, we have various viable locations to choose from. Once the trucks leave their respective DCs with loaded commodities and drones on board, deliveries commence when authorized drop-off sites are reached.

Drones fly out and make deliveries beyond customer-requested destinations using collected co-ordinates on Euclidean planes as waypoints. These designated drop-off points are referred to as demand nodes situated along this path where deliveries take place before moving forward according to our protocols based on mod-determined capacity requirements while maintaining compliance. The trucks get unloaded at each node until all necessary requests at designated points have been fulfilled precisely as per our procedures. The drones then return for recharging processes before deploying again until every request is completed in full concordance with our regulations while making sure not to exceed their capabilities or limitations under any circumstances. Finally, either returning or proceeding further based on authorization completes the process.

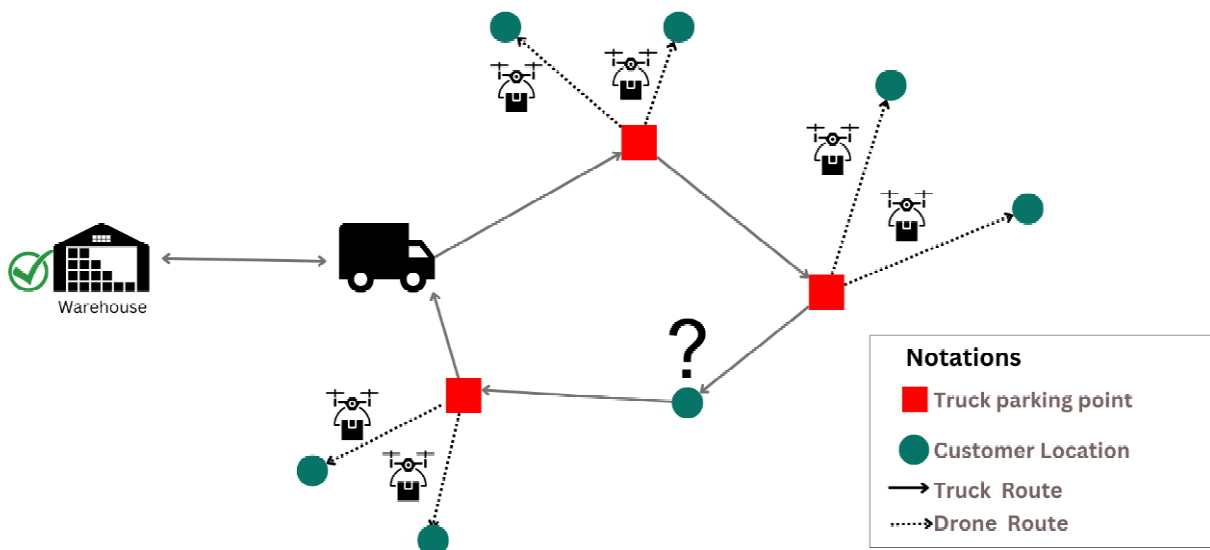


Figure 3.1: The truck-drone delivery diagram.

Chang and Lee (2018) explored a comparable operating method for a single-facility distribution operation, as seen in Figure 1.1.

3.2 Difference between CVRP and Truck-Drone Hybrid Model

There are two possible alternatives to improve delivery logistics: the truck drone hybrid last mile delivery model and the capacitated vehicle routing problem that utilizes trucks for delivery. Despite relying on trucks each model offers distinct advantages and disadvantages. However, what sets apart the hybrid truck drone model is its incorporation of drones. According to Dorling et al.’s work (2017) this variation impacts both how the challenge is framed and what goals and constraints need to be considered.

### 1. Problem Structure:

According to Toth and Vigo (2014), efficient delivery in CVRP involving trucks hinges upon three vital elements - a depot, fleet of uniform trucks, and several clients each with specific requirements. The main objective is to minimize delivery time due to vehicle carrying capacity constraints as pointed out by Eksioglu et al., (2009). This encompasses trip duration as well as actual disbursement time for goods during deliveries. In contrast, Murray & Chu's (2015) hybrid truck-drone technology uses vehicles plus drones for last-mile deliveries. Customer orders are fulfilled using this two-pronged approach comprised of vehicles plus aerial devices. Accordingly, Dorling et al.'s (2017) model comprises depot, a group of uniformed vehicles, drones operating within certain flight limits, and different assortments specified by the respective customers. Trucks perform like mobile drone depots thus allowing quadcopters to be released or recovered during transportation. Agatz et al. (2018) assert that this strategy utilizing both vehicles and drones allows minimizing the overall delivery times.

### 2. Objective function:

Ensuring a more efficient delivery system is the key aim for both strategies. In integrating both trucks and drones, however, one cannot overlook other parameters that contribute significantly to this outcome. The objective function employed by hybrid models would necessarily have to take into account time devoted to customer service activities or periods in which vehicles or their air-bound counterparts travel distance (Murray & Chu, 2015). Coordination is essential at all levels within these models, taking into account issues like drone-launch/retrieval duration times and waiting periods encountered by trucks while drones complete their deliveries (Agatz et al., 2018).

### 3. Restrictions:

To guarantee that each client's specific needs are met and prevent overloading of delivery vehicles, both traditional trucks and modern drone-truck hybrid systems have limitations. However, employing drones as part of this system introduces further restraints. Dorling et al. (2017) highlight these constraints which not only encompass drone range but also entail seamless coordination between the truck and drone while considering their respective speeds.

### 4. Complications:

Compared to the truck-drone hybrid model, the CVRP with trucks provides a more straightforward solution by eliminating extra coordination between different vehicles. As highlighted by Agatz et al. (2018), implementing a truck-drone system requires introducing new decision criteria and limits to capture their interactions effectively. Legal considerations further complicate finding solutions for prospective issues posed by these systems. Such complexities include legal restrictions on drone operations such as flying height limitations, no-fly zones, and line-of-sight requirements as noted by Dorling et al., (2017).

### 5. Solution approaches:

With the difficulty level increasing in operating a truck-drone hybrid setup effectively tackling problems may demand out-of-the-box thinking. That said, applying sophisticated methodologies like metaheuristics, decomposition strategies or machine learning based approaches could offer further insight into optimizing this novel technology (Eksioglu et al., 2009; Agatz et al., 2018). Furthermore, it's important to note that traditional optimization mechanisms, for example linear programming and mixed-integer programming remain customary procedures to enhance both CVRP with trucks and truck-drone hybrids.

### 6. Effectiveness and efficiency:

The combination of trucks and drones within the hybrid model displays exceptional efficacy through its ability to optimize shipping times while enhancing overall efficiency levels significantly. This sophisticated technique incorporates drone utilization during "last-mile" deliveries by leveraging their speed/mobility attributes when conventional truck-only shipments are impeded due to barriers like dense traffic or difficult terrain (Murray & Chu, 2015). Furthermore, overcoming transportation challenges pertaining to low accessibility remote areas can be a straightforward task when utilizing drones to transport items as opposed to conventional vehicles. Based on Agatz et al.'s research (2018), Hybrid truck-drone interventions can prove to be significantly more time-efficient and economical compared to the existing CVRP, which uses cars as their primary delivery mode. Consequently, implementing a hybrid model may lead to higher levels of customer satisfaction whilst simultaneously reducing operational expenses.

### 7. Sustainability:

Taking steps to create a hybrid model of truck drone could pave the way for environmentally responsible logistical operations. The findings of Dorling et al.'s research from 2017 indicate that merging trucks with drones' results in lower fuel usage and carbon emissions during deliveries particularly when electrically powered drones are used. Furthermore, utilizing drones for last mile deliveries could ease traffic congestion by reducing overall truck presence on roads as highlighted by Agatz et al.'s (2018) research – which could impact positively both the environment and quality of life in urban areas.

### 8. Difficulties and restrictions:

Potential advantages concerning drone deliveries are recognized by Bamburly (2015), but associated challenges require intricate models and advanced problem-solving techniques. Novel legal constraints apply solely to drone actions. As one of the pioneering nations utilizing drone technology for deliveries, we focus on the United Kingdom as Clothier et al. (2015) did in illustrating such challenges. The Civil Aviation Authority (CAA) (2020) in charge of overseeing airspace activity established regulations geared towards ensuring overall safety and reducing possible threats that could arise. Efficiency decline regarding hybrid truck-drone systems is affected by specific clauses such as the maximum altitude limit restrictions alongside no-fly zones and line-of-sight requirements. In agreement with CAA regulations cited by Rahman et al. (2017), it is illegal for drones to exceed heights above 400 feet (120 meters) within UK airspace. To minimize collision incidents between manned and unmanned aircraft, these limitations have also been put in place, resulting in limited accessibility to vertical areas during delivery tasks that require navigation around tall structures and terrain (Choi et al., 2019). One key consideration when it comes to operating drones is CAA-designated "no-fly zones" where it is expressly forbidden to use these devices (CAA, 2020). These locations typically cover prisons, military sites, and are quite congested spaces such as stadiums or urban areas--sites where there may be large public gatherings according to Huang et al. (2018). These restrictions possibly reduce the number of available routes making practical drone delivery difficult as per Dorling et al. (2017).

Another non-negotiable regulation relates to direct line-of-sight requirements between operators and their drones; the CAA has prescribed the necessity for an always-visible visual connection over short distances within 500m horizontally & 400 feet vertically according to their estimation (CAA,2020). This ensures necessary safety checks are executed and possible emergencies limited effectively notes Rahman et al. (2017). Nevertheless, this allowable range could likely cause significant problems in environments where buildings or high-density human activity makes it tough for operators to maintain sighting as stated in Choi et al.'s study (2019).

Finally, another important factor affecting drone operation in the UK is its famous rainfall with weather patterns notoriously shifting unexpectedly (Yoo & Kim, 2017) affecting flight capability significantly even with smaller models of drones per Dorling et al.'s findings (2017). Resulting cancellations or delivery delays could therefore occur due to such weather conditions. When evaluating the use of drones for delivery purposes, one must consider not just legislation or environmental impacts but also technological obstacles such as battery life limitations and cargo capacity; Levin et al.'s study supports this claim (2020). As Bamburly notes (2015), constrained weight capacity combined with short flying times restricts both range and items capable of being transported via drone which consequently limits potential applications further. Another consequential factor often discussed is privacy rights affecting individuals which may present difficulties working with drone deployment, namely noise pollution, invasion of privacy or possible concerns with public reception according to Clarke's research (2014); Nhan et.al(2020) also highlights this matter. Employing a truck-drone hybrid model for distribution purposes only succeeds if these roadblocks are carefully and thoughtfully managed per Clothier et al. (2015). However, regulations set by the civil aviation authority of the UK alongside unique technological limitations and social anxieties necessitate innovative solutions through collaboration between government entities including corporate representatives engaging with public concerns and feedback-according to Huang et al. (2018).

### 9. Technical limits:

The existing restrictions on payload volume, time of flight, and battery life need to be taken into consideration as drone technology for delivery continues to progress. This indicates that there may be challenges involved in the effective deployment of a hybrid delivery approach including trucks and drones. In addition, Murray and Chu (2015) have expressed worry over potential threats to public safety, including collisions involving several drones, theft, and invasions of personal privacy. In order to overcome these challenges, fresh methodologies for modelling and problem solving, as well as reliable frameworks for operating norms, are presented as potential solutions. Even while the truck-drone hybrid model is comparable to truck deliveries and delivery operations that use trucks and drones, the presence of drones introduces a number of major distinctions (Murray & Chu, 2015). Although this approach has the

potential to achieve increases in efficiency as well as faster delivery times, it is still in need of more sophisticated solutions in order to overcome the challenges it faces. According to Agatz et al. (2018), assessing the possibility of a truck-drone hybrid delivery system is worthwhile since it has the potential to revolutionise last-mile deliveries and contribute to sustainable logistics as drone technology continues to develop in a way that satisfies regulatory standards.

### 3.3 Assumptions

Each node has been given x and y coordinates and is located on a Euclidian plane. It is anticipated that all the trucks in the fleet will have the same maximum travel speed, package-carrying capacity, drone loading space, and operational expenses. Just one package can be transported by each drone at a time. As a result, a drone would only visit one J node for every dispatch with the same operating cost and speed. This study does not consider the difference in travel costs between loaded and unloaded trucks and drones. After completing their routes, trucks and drones return to their respective starting sites. At the start of the planning horizon, a fleet of trucks and drones is available, and there are no maintenance or breakdown constraints. Both trucks and drones travel at constant speeds that are unaffected by traffic or weather conditions. Truck and drone operations costs are consistent, and there are no additional costs connected with customer locations or specific time periods.

#### 3.3.1 General Assumptions

All parcels are considered to be the same in terms of weight and dimension. If a customer needs more than one parcel, their location is represented by multiple nodes with the same coordinates that overlap. A truck will not make the delivery directly to the customer, deliveries will be made by the drone to the customer from the truck parking point or distribution center. Drones can't go straight from the distribution centers (D nodes) to the J nodes. All unmanned aerial vehicles or drones must be initiated from trucks at K nodes. Trucks and drones can reach all customer sites, and there are no constraints on truck parking points for drone deployment. Drones fly in straight lines from truck parking areas to consumer locations. Drone activities are not subject to any legal or regulatory limits. There is no limit to how many times a drone can be launched from a truck along a delivery route because demand for each customer is known ahead of time and does not fluctuate during the planning horizon. Except for the loading and dispatching of drones at truck parking spots, there is no collaboration or interaction between trucks and drones.

To summarize, these assumptions simplify the problem by focusing on minimizing overall delivery time while taking operational restrictions like capacity and flying range into account. They also give a framework for coordinated truck and drone routing, making it easier to model and solve the problem.

#### 3.4 Mathematical model of Truck only VRP

The following is a mathematical formulation of the Vehicle Routing Problem (VRP) with trucks for delivery using Euclidean distance, with the goal of minimizing the total delivery time. Accordingly, Souza, I.P., Boeres, M.C. and Moraes, R.E. (2023),

Calculating the Euclidean distance between two nodes, i and j, looks like this:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Decision variables:

$$x_{ij}^k = \{1, \text{if truck travels from node } c_i \text{ to node } c_j\} \text{ otherwise, } 0$$

Input Parameters:

$N$  = customers number

$K$  = maximum number of trucks

$Q$  = maximum capacity of each truck

$d_{ij}$  = distance from nodes  $c_i$  and  $c_j$ , with  $d_{ij} = d_{ji} \forall i, j \in \{0, 1, 11, \dots, N\}$

$q_i$  = demand of nodes  $c_i \in V$

Objective Function:

$$\min z = \sum_{i=0}^{60} \sum_{j=0}^{60} \sum_{k=1}^9 d_{ij} x_{ij}^k$$

The aim of this model is to minimize the total distance which will lead to minimizing the total delivery or operation time as well.

Subject to:

$$\sum_{k=1}^9 \sum_{i=0}^{60} x_{ij}^k = 1, j = 1, 2, \dots, 60 \tag{1}$$

$$\sum_{k=1}^9 \sum_{j=0}^{60} x_{ij}^k = 1, j = 1, 2, \dots, 60 \tag{2}$$

$$\sum_{i=0}^{60} x_{ij}^k - \sum_{j=0}^{60} x_{hj}^k = 0, k = 1, 2, \dots, 9; h = 1, 2, \dots, 60 \tag{3}$$

$$\sum_{j=0}^{60} q_j \left( \sum_{i=0}^{60} x_{ij}^k \right) \leq Q, k = 1, 2, \dots, 9 \tag{4}$$

$$\sum_{j=1}^{60} x_{0j}^k \leq 1, k = 1, 2, \dots, 9 \tag{5}$$

$$\sum_{i=1}^{60} x_{i0}^k \leq 1, k = 1, 2, \dots, 9 \tag{6}$$

$$x_{ij}^k \in \{0,1\}, i \neq j; j = 0, 1, \dots, 60; k = 1, 2, \dots, 9 \tag{7}$$

The Capacitated Vehicle Routing Problem (CVRP), in which a fleet of trucks is tasked with delivering goods to a collection of clients, is represented mathematically by this model. Whether a truck "k" moves from node "c<sub>i</sub>" to node "c<sub>j</sub>" is indicated by the decision variable "x<sub>ij</sub><sup>k</sup>". x<sub>ij</sub><sup>k</sup> equals 1 if it does, 0 otherwise. The objective function is to minimize the overall distance, or alternatively, the total delivery time, which is calculated by adding the distances 'd<sub>ij</sub>' between each pair of nodes 'i' and 'j' and 'x<sub>ij</sub><sup>k</sup>' (indicating whether truck 'k' takes that route or not).

There are a number of restrictions on this model:

1. The first constraint ensures that each consumer 'j' is visited by each truck exactly once. Each client node is entered from exactly one other node by one truck, as determined by the sum of all nodes (i) and trucks (k) for the formula "x<sub>ij</sub><sup>k</sup>".
2. The second constraint ensures that each consumer 'i' gets left by one of the trucks exactly once. Each client node is left to exactly one other node by one truck, as shown by the requirement that the sum of all nodes (j) and all trucks (k) for the expression "x<sub>ij</sub><sup>k</sup>" equals 1.
3. The third constraint ensures that a truck must exit from a customer node if it enters that node, keeping the truck's route continuous.
4. Constraint (4) assures that the total demand of all clients served by a truck 'k' does not exceed the truck's maximum capacity 'Q'.
5. Each vehicle may only depart from node 0 and return to the depot node 0 once due to restrictions (5) and (6).
6. A binary limitation known as constraint (7) mandates that "x<sub>ij</sub><sup>k</sup>" can only take the values 0 or 1, assuring that a truck will either take or not take a route between two nodes.

In summary, this mathematical model aims to determine the best routes for a fleet of trucks to deliver goods to a set of customers such that the total distance (or time) is minimised, each customer is only visited once, the total demand served by each truck does not exceed its capacity, and each truck only departs from and returns to the depot once.

**3.5 Mathematical Model of Truck-drone VRP**

The suggested Mixed-Integer Linear Programming (MILP) model is described in more detail in Moshref-Javadi et al.,

2020. This mathematical model aims to minimize the total time of delivering goods using a single truck and multiple drones from a single depot or warehouse. This problem can be described by a mixed-integer linear programming (MILP) model.

Let,

$i, j \in N$ : Indices representing customer locations and the warehouse, where  $N = \{0, 1, \dots, n\}$ , 0 represents the warehouse and  $n$  represents the number of customer

$k \in K$ : Index representing trucks, where  $K = \{1, \dots, m\}$  is the available trucks

$l \in L$ : Index representing drones, where  $L = \{1, \dots, p\}$  is the set of available drones

Parameters:

$t_{ijk}$ : Time needed to travel from  $i$  location to  $j$  location using  $k$  number trucks

$d_{ijl}$ : Time required for drone  $l$  to travel from  $i$  location to  $j$  location

$q_i$ : Customer demand

$Q_k$ : Truck capacity

$W_l$ : Drone capacity

$R_l$ : Maximum flight range of drone  $l$

Decision variables:

$x_{ijk} \in \{0,1\}$ : binary variable equal to 1 if truck  $k$  travels from  $i$  location to  $j$ , otherwise 0

$y_{ijl} \in \{0,1\}$ : binary variable equal to 1 if drone  $l$  travels from  $i$  location to  $j$ , otherwise 0

$w_i$ : Continuous variable representing the time of the delivery at customer location  $i$

Objective function:

Minimize the total delivery time:

$$\min \sum_{i,j}^N \sum_k^K \sum_l^L t_{ijk} \times x_{ijk} \times y_{ijl}$$

Constraints:

Each customer location must be visited once by a truck or a drone:

$$\sum_k^K x_{ijk} + \sum_j^N y_{ijl} = 1, \forall i \in N \setminus \{0\}$$

Truck flow conservation:

$$\sum_j^N x_{ijk} - \sum_j^N y_{ijk} = 0, \forall i \in N, \forall k \in K$$

Drone flow conservation constraints:

$$\sum_j^N y_{ijl} - \sum_j^N y_{jil} = 0, \forall i \in N, \forall l \in L$$

Truck capacity:



$$\sum_{i,j}^N q_i \times x_{ijk} \leq Q_k, \forall k \in K$$

Drone capacity:

$$\sum_{i,j}^N q_i \times y_{ijl} \leq W_l, \forall l \in L$$

Maximum flight range of drone:

$$\sum_{i,j}^N d_{ijl} \times y_{ijl} \leq R_l, \forall l \in L$$

Truck time synchronization:

$$u_i + t_{ijk} - M \times (1 - x_{ijk}) \leq u_j, \forall i, j \in N \setminus \{0\}, \forall k \in K, \text{ where } M \text{ is a large constant}$$

Drone time synchronization:

$$u_i + d_{ijl} - M \times (1 - y_{ijl}) \leq u_j, \forall i, j \in N \setminus \{0\}, \forall l \in L, \text{ where } M \text{ is a large constant}$$

Non-negativity and integrality constraints:

$$x_{ijk} \in \{0,1\}, \forall i, j \in N, \forall k \in K, y_{ijl} \in \{0,1\}, \forall i, j \in N, \forall l \in L, u_i \geq 0, \forall$$

- (1) A truck or a drone needs to make one trip to each of the customer locations at least once.
- (2) The conservation of truck flow assures that the number of trucks coming into and going out of a site is equal.
- (3) The method of "drone flow conservation" assures that an equal number of drones are entering and exiting a region.
- (4) The limitations placed on the capacity of trucks make certain that the entire demand met by a truck does not exceed the capacity of the truck.
- (5) Capacity restrictions imposed on drones guarantee that the entire demand that can be satisfied by a drone does not go beyond its capabilities.
- (6) The limitations placed on a drone's flight range ensure that the maximum distance it can travel does not exceed the entire distance it can cover.
- (7) The process of time synchronization for trucks guarantees that the delivery time at location j will be more than or equal to the delivery time at location i plus the travel time between locations i and j.
- (8) The process of time synchronization for drones assures that the delivery time at location j will be more than or equal to the delivery time at location i including the trip time between locations i and j.
- (9) The non-negativity and integrality restrictions make certain that the choice variables are assigned the correct values.

In simple terms, the purpose of this model is to determine the most efficient path that can be taken by a fleet of trucks and drones to transport goods to various customer locations, with the goal of reducing the total amount of time required for delivery while considering a variety of operational restrictions such as capacity and flight range.

### 3.6 Model Implementation

The central aim here is providing an insightful review of how Truck-Drone Hybrid Last Mile Delivery (TDHLM) works from an operational perspective. As its title suggests, TDHLM uses both trucks and drones together in order deliver goods straight into consumers' hands without extra hassle. This method is novel-and forward-thinking that grapples with age-old problems concerning last-mile delivery logistics expeditiously yet accurately. The creation

process of TDHLMMD enveloped a combination of Python programming language and Gurobi optimization software for the most efficient output results possible.

### 3.6.1 Computational Capabilities

The operation of the truck-drone hybrid last-mile delivery model successfully took place on an appropriately equipped desktop computer running Gurobi Optimizer Tool without hitches. It leverages Windows 11 operating systems' features - renowned for their efficiency to support many applications like Python programming language seamlessly - making it ideal for use within this context.

To achieve fast and accurate computation concerning optimization challenges encountered while developing this model, we applied the Intel(R) Core (TM) i5-7600K CPU @ 3.80GHz. The system's processor comes with the advantage of a rapid clock speed and extensive processing capabilities.

Multitasking during model management was efficient, thanks to the 16 gigabytes RAM capacity present in our desktop computer system. Even during peak processing periods, we observed consistent performance without any possible risk of slowdowns or stuttering.

Conclusively, when utilizing Gurobi Optimizer's capabilities alongside Python - thanks to our powerful hardware systems' processing power - we effectively achieved streamlined development and implementation procedures necessary for our truck-drone hybrid last-mile delivery model. Our system's tight specifications ensure efficient control over computational demands, handling massive data at optimal speed while delivering reliable results consistently.

### 3.6.2 Parameter Definition

Before deploying the Truck-Drone Hybrid Last Mile Delivery (TDHLMMD) model, one must specify crucial parameters that will determine how it operates most effectively:

1. Customer location volume is represented by "nc" and determines the number of consumers needing product delivery – more clients increase logistical difficulty exponentially when identifying feasible routes.
2. The total number of trucks available for use must also be determined using "nm" – this data allows for proper resource allocation through employing mobile storage units from where drones will dispatch strategically towards various points in transit along any given route during product distribution.
3. It is recommended that users set a maximum drone count with "pn" to further improve delivery efficiency while avoiding traffic congestion during transport, ultimately leading to faster product receipt timeframes for customers.
4. Staying informed about customer demand levels with "q" becomes an essential component in understanding how many trips trucks and drones alike must make in order to fulfill all client requirements necessary - assessing this metric explicitly informs decisions based on logistics management strategy around meeting operational goals most efficiently.
5. Resource management considerations regarding limitations inspired by drone flying range ("DR"), drone capacity restrictions ("CD"), and truck carrying capacity limits ("TQ") must be taken into account to ensure that the model can be employed in practice.
6. Finally, in developing transportation solutions through this model, it is critical first to collect accurate geographical coordinates for both customer and depot locations – these data are essential in calculating travel distances between points effectively and estimating transportation times accurately.

### 3.6.3 Calculation of the Distance Matrix

Once the parameters of the issue have been defined, the distance matrix may be calculated. The Euclidean distances between every possible pair of places are recorded and kept in a two-dimensional array called the distance matrix. Using the coordinates of each place, the corresponding distances may be determined. It is expected that the speeds of trucks and drones (both capitalised) will be directly proportional to the distances shown above.

### 3.6.4 Model Development

Developing a mathematical model precedes executing any plan. The formulation process goes through different stages involving choice variables declaration, ideal functions establishment with additional constraints inclusion. Within this context, truck coordinates as variable "x" and drone coordinates as variable "y" play crucial roles in directing where goods or services should be delivered to all customers effectively while considering other parameters.

To ensure effective decision making in achieving reduced delivery time coupled with decreased cost through

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efficiency in movement, appropriate objective functions which would compute both total travel time for every route taken included with sums of choices of mentioned versatile variables is established.

Restrictions level set on such models include precise visitation to customer locations only once by one means (either truck or drone), always avoiding overload on usable capacity for both types of vehicles maintaining consistency with these restrictions while ensuring range expectations adequately managed - ultimate aim being management efficiency as regards hybrid delivery.

### 3.6.5 Model Improvement

After creating a complete mathematical model, optimization can officially start. Utilizing Gurobi optimizer's advanced mathematical programming techniques and heuristics, a solution can be found for even the toughest of scenarios. Given their demonstrated capacity in handling large scale optimization problems efficiently, Gurobi was chosen as our specialized software for this project. Our resulting optimization method grants us optimal values for decision variables, namely the most economical routes taken by our drones and trucks. An important facet of this process we must constantly bear in mind is ensuring accurate input parameters reflecting demand values and distance matrix- fundamental parts of generating a high-caliber solution.

### 3.6.6 Interpreting and Visualizing the Solution

A comprehensive understanding of truck and drone routes is essential to interpreting our solution accurately for every client location's servicing. The values assigned to the choice variables  $x$  and  $y$  are utilized to gain this necessary information. Whenever the values of certain choice variables are 1 such as  $x[i,j,k]$  or  $y[i,j,l]$ , direct traveling routes can be taken through truck  $k$ , or drone  $l$  from place  $i$  for transportation towards location  $j$  is feasible. Our dependency on the  $u$  variable stems from it providing useful information about loads at each site, which makes it easier for us to determine how much carrying capacity each vehicle uses currently. In order to fully grasp our solution's final outcome, having a clear perception of these delivery paths is vital. Therefore, using Python's matplotlib graphing program aided us in creating graphical representations for both truck and drone routes present in our model. Depots were represented using their own unique color shades, while locations appeared as points over this scatter plot within two dimensions. Color-coded or differently styled lines connected locations representing trucks' or drones' traversal based on their mode.

### 3.6.7 Validation and testing of the model

Model validation and testing are important elements in the process of putting a model into action. Testing the model on real or simulated datasets is necessary in order to guarantee that it generates results that are consistent with logic and correspond to what was predicted. For instance, in order to test the model, one may check to see if the whole demand is within the capability of the truck and the drone. In addition, it is possible to evaluate the scalability and resilience of the model by applying it to more challenging scenarios and putting it through an evaluation.

### 3.6.8 Restrictions and Hypotheses

While the Truck-Drone Hybrid Last Mile Delivery (TDHLMD) model provides an innovative resolution for last-mile transportation problems, there are certain limitations and assumptions to consider before implementation. One crucial factor is that the TDHLMD assumes all locations are accessible by both trucks and drones - which may not be a practical assumption in all cases. This constraint requires focus when addressing.

Moreover, the immediate demands of freight scheduling can impact negatively when concerning natural hindrances like weather conditions or traffic flow. These challenges - along with vehicle breakdowns- do not have consideration within this specific perspective. This framework accepts conditional determinism focused on client-location-demand - however occurrence of unexpected or fluctuating demand further complicates these circumstances wholly.

The execution here requires distinct processes like parameter specification, distance calculation, model formulation, optimization, solution interpretation and visualization including model validation and testing. Implementing the TDHLMD model provides a foundation for understanding as well as improving hybrid truck-drone last-mile delivery systems - on the other hand- limitations must be taken into consideration when applied to practice.

There is potential to improve the TDHLMD model in real-world scenarios by integrating limits and uncertainties inherent within such settings. By doing so, we can increase its practicality and usefulness.

## 4. Data Collection

### 4.1 Dataset

**Table 1:** List of the physical parameters by MACO

Parameter	Description	Value	Unit
$F_{drone}$	Fixed cost per drone used	3.0769230	\$/drone/truck/day
$F_{truck}$	Fixed cost per truck used	20.855944	\$/truck/route/day
$c_d$	Drone travel cost	0.0000311	\$/meter
$c_t$	Truck travel cost	0.0027968	\$/meter
$c_{t-w}$	Truck idle waiting cost	0.0059722	\$/second
h	Number of drones per truck	3	-
$v_{dr}$	Average drone speed	16	meter/second
$v_t$	Average truck speed	7	meter/second
$s_i$	Truck parking time at each customer node	90	second
$t_{dr-unload}$	Time required for a drone to unload a parcel	15	second
$t_{dr-load}$	Time required to load a parcel onto a drone and dispatch it	20	second
$Q_{max}$	Truck capacity	10,30	parcel/truck
$k_{drone}$	Circuitry factor of drone travel	1	-
$k_{truck}$	Circuitry factor of truck travel	2.46	-

The set of parameters used in the Multi-Ant Colony Optimization (MACO) were adapted from the research done by Ting and Chen (2013). Each value in the table corresponds to a significant component of the model:

The fixed costs ( $F_{truck}$  and  $F_{drone}$ ) show the ongoing expenditures associated with using trucks and drones for deliveries. These must be taken into consideration in any cost-benefit analysis because they greatly increase operational costs.

The variable costs connected to each unit distance travelled by the truck and the drone, respectively, are accounted for by the travel costs ( $c_d$  and  $c_t$ ).

The cost paid when the truck is idle while waiting for the drone to return is known as the truck idle waiting cost, which can have an effect on the system's overall efficiency.

To determine delivery timings and map out the best routes, it is essential to know the average speed of the truck and the drone ( $v_{dr}$  and  $v_t$ ).

The amount of time needed to load and unload packages from the drone ( $t_{dr-unload}$  and  $t_{dr-load}$ ) is also crucial for determining delivery times and assessing the effectiveness of the system.

The number of deliveries that can be handled in a single trip depends on the truck's capacity ( $Q_{max}$ ) and the number

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of drones per truck ( $h$ ), which impacts the system's service capacity.

The indirectness of each vehicle's tracks is accounted for by the circuitry factors for the drone and the truck ( $k_{drone}$  and  $k_{truck}$ ), which is crucial when determining routes in urban areas with complicated infrastructure.

The MACO can precisely simulate and optimise a truck-drone hybrid delivery system by incorporating these factors, helping logistics organisations make informed choices about putting such systems in place.

**Table 2:** A summary of the capabilities and published specifications of delivery drones (from facility to destination directly)

Company	Speed (km/h)	Flight Range (km)	Endurance (minutes)	Payload	Size (m)	References
Company A	161	12	30	2.3kg	1.8m	(Bloomberg,2019) (Raconteur, 2018)
Company U	-	-	30	4.5kg	-	(UPS, 2017)
Company D	130	32.5	40	4kg	1.8m	(DHL,n.d.)
Company W	72	-	-	0.6 ft3	-	(workhorse, n.d.)
Company UE	48	9.7	-	2 meal bags	-	(Hawkins, 2019)
Company G	112	20	-	1.5kg	1.3m	(Wing, n.d.)
Company R	-	-	15	2kg	-	(Nikkei Asian Review, 2019)

Organizations like Amazon, DHL, UPS, Workhorse, Uber Eats, Rakuten & Wing have distinctively unique methods when it comes to deploying drones with regard to delivery. These techniques and information on critical factors such as endurance, payload, flying range, speed etc. are part of an extensive dataset that we now examine. We believe that understanding the differences between companies operating in the drone delivery arena is critical for assessing levels of performance offered by each system. While examining this data, it's essential to acknowledge that certain businesses may not have accurate records of all these crucial aspects, creating gaps which make exhaustive and objective face-offs amongst varied drone delivery systems challenging.

Our focus at this juncture is on Company A which provides an extensive collection of information on their drones' attributes such as sizes, speeds, flying ranges, endurance, and cargo capacities. We have focused here due to several reasons including access to a wealth of data as our investigation becomes more comprehensive and exhaustive. Likewise, Company A, being perceived as leaders in developing innovative drone tech solutions for delivery services makes them a valuable resource with substantial body-of-data to draw upon. The significance of making use Company A's data in our study is inherently tied to optimizing a hybrid truck-drone delivery system with our novel algorithms and approaches. These methodologies hold the potential for benefits such as overcoming some limitations imposed by drones' shorter flying radius while benefiting from automobiles' flexibility which could reduce transportation costs substantially. Gurobi Optimizer is an exceptional mathematical programming solution renowned for unparalleled speed and efficacy. The accuracy of our simulation of Amazon's delivery system using Gurobi and access to their extensive dataset allows us to identify the most beneficial alternatives within our reach. Given its expansive scale of operation, Company A requires highly effective yet cost-efficient delivery systems, making our approach even more valuable beyond digital space. Our methodology enhances efficiency within existing drone-delivery operations implemented by Company A while providing additional possibilities for future logistics research. By utilizing Company, A 's data as a foundation in modeling optimization procedures, we establish a suitable framework which can be applied across other logistical scenarios from various industries. Not only is it sensible but also tactical to leverage on information offered by Company A towards expanding relevant investigations in this field.

## 5. Result and Analysis

This chapter focuses on presenting and analyzing the results derived from our mathematical modeling conducted earlier by utilizing Gurobi Optimizer as a platform this study aimed at finding an effective resolution for last-mile delivery concerns with minimal investment involved, satisfying key operational requirements in due process; thus, devising a hybrid truck-drone approach.

Our aim was to lower overall delivery time incurred while maintaining the required functionality standards. Here's what we've obtained through our optimization process:

"The optimal solution has been found (tolerance 1.00e-04)."

Best objective 3.246036597388e+00, best bound 3.246036597388e+00, gap 0.0000%

```

Gurobi Optimizer version 10.0.1 build v10.0.1rc0 (win64)

CPU model: Intel(R) Core(TM) i5-7600K CPU @ 3.80GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 4 logical processors, using up to 4 threads

Optimize a model with 454 rows, 1365 columns and 6828 nonzeros
Model fingerprint: 0x505de39e
Variable types: 13 continuous, 1352 integer (1352 binary)
Coefficient statistics:
  Matrix range    [8e-02, 4e+01]
  Objective range [8e-02, 9e-01]
  Bounds range    [1e+00, 1e+00]
  RHS range       [1e+00, 3e+01]
Presolve removed 144 rows and 1029 columns
Presolve time: 0.01s
Resolved: 310 rows, 336 columns, 2016 nonzeros
Variable types: 12 continuous, 324 integer (324 binary)
Found heuristic solution: objective 10.6070475

Root relaxation: objective 2.037161e+00, 46 iterations, 0.00 seconds (0.00 work units)

Nodes | Current Node | Objective Bounds | Work
Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----
  0 0 | 2.03716 0 20 | 10.60705 2.03716 80.8% | - 0s
H 0 0 | 7.0933686 2.03716 71.3% | - 0s
  0 0 | 2.15213 0 25 | 7.09337 2.15213 69.7% | - 0s
  0 0 | 2.15213 0 25 | 7.09337 2.15213 69.7% | - 0s
H 0 0 | 4.9128134 2.15213 56.2% | - 0s
H 0 0 | 3.8746434 2.17493 43.9% | - 0s
  0 0 | 2.17493 0 23 | 3.87464 2.17493 43.9% | - 0s
  0 0 | 2.20146 0 22 | 3.87464 2.20146 43.2% | - 0s
H 0 0 | 3.7402216 2.20146 41.1% | - 0s
  0 0 | 3.74022 2.20146 41.1% | - 0s
H 0 0 | 3.6179790 2.20146 39.2% | - 0s
  0 0 | 2.20146 0 20 | 3.61798 2.20146 39.2% | - 0s
  0 0 | 2.20146 0 20 | 3.61798 2.20146 39.2% | - 0s
H 0 0 | 3.3682791 2.20146 34.6% | - 0s
  0 0 | 2.20146 0 20 | 3.36828 2.20146 34.6% | - 0s
  0 0 | 2.20146 0 20 | 3.36828 2.20146 34.6% | - 0s
  0 2 | 2.20146 0 20 | 3.36828 2.20146 34.6% | - 0s
H 1059 562 | 3.2460366 2.40760 25.8% | 11.0 0s

Cutting planes:
  Learned: 6
  Gomory: 13
  Cover: 1
  Implied bound: 3
  MIR: 13
  Flow cover: 27
  Zero half: 4
  RLT: 40

Explored 5046 nodes (56684 simplex iterations) in 1.34 seconds (1.27 work units)
Thread count was 4 (of 4 available processors)
    
```

Figure 5.1: Gurobi optimizer solution process.

It took the approach 5046 node explorations and 56684 simplex iterations in 1.34 seconds to arrive at this result, which is indicative of the computational difficulties of the problem. The fact that the optimisation was completed so rapidly, despite the complexity of the problem, is evidence of both the efficiency of the Gurobi optimizer and its ability to solve difficult logistical issues.

### 5.1 Initial model analysis

Initially, I utilized version 10.0.1 of Gurobi Optimizer on my personal computer with an Intel Core i5 CPU. The model's first iteration comprised 1365 columns indicating decision variables and 454 rows representing constraints that needed to be met. There was a total of 6828 non-zero items indicating connections between these limitations and variables, both continuous and integer, the latter including binary variables. The complexity inherent in this model underscores the challenges associated with last-mile delivery scenarios, in which hybrid solutions like truck-drone combinations may be utilized. Employing advanced optimization tools like Gurobi is crucial for addressing such intricate issues.

### 5.2 Process of Optimisation

After taking a brief break from our findings, the first thing we need to do is gain an appreciation for the complexities that are involved in optimising our models by making use of the tools provided by the Gurobi Optimizer. These techniques include presolving, heuristics, cutting planes, and the branch-and-bound strategy. Before beginning work on the primary optimisation phases, our presolving strategy removed 132 rows and 1005 columns, which significantly cut down on the complexity of the problem. These approaches made our model simple to solve while simultaneously saving a significant amount of time.

The application of heuristics led to the discovery of various potential solutions, some of which may not have been ideal, but which were nonetheless important indicators for the performance measurement of hybrid delivery systems across optimised processes.

By utilising cutting planes such as linear inequalities intertwined with branch-and-bound methodology, suboptimal areas were easily identifiable within the resulting analytics reports, making impressions more accurate through streamlined investigation tactics. This was accomplished by splitting larger solution spaces into smaller portions.

### 5.3 Evaluation of the Data

The primary goal of our optimisation challenge was to cut down on the objective function, which in this instance reflected the total delivery time that related to operating the hybrid truck-drone delivery system. Before beginning the optimisation process, Gurobi came to the conclusion that the goal function should have a value of 10.7331389e.

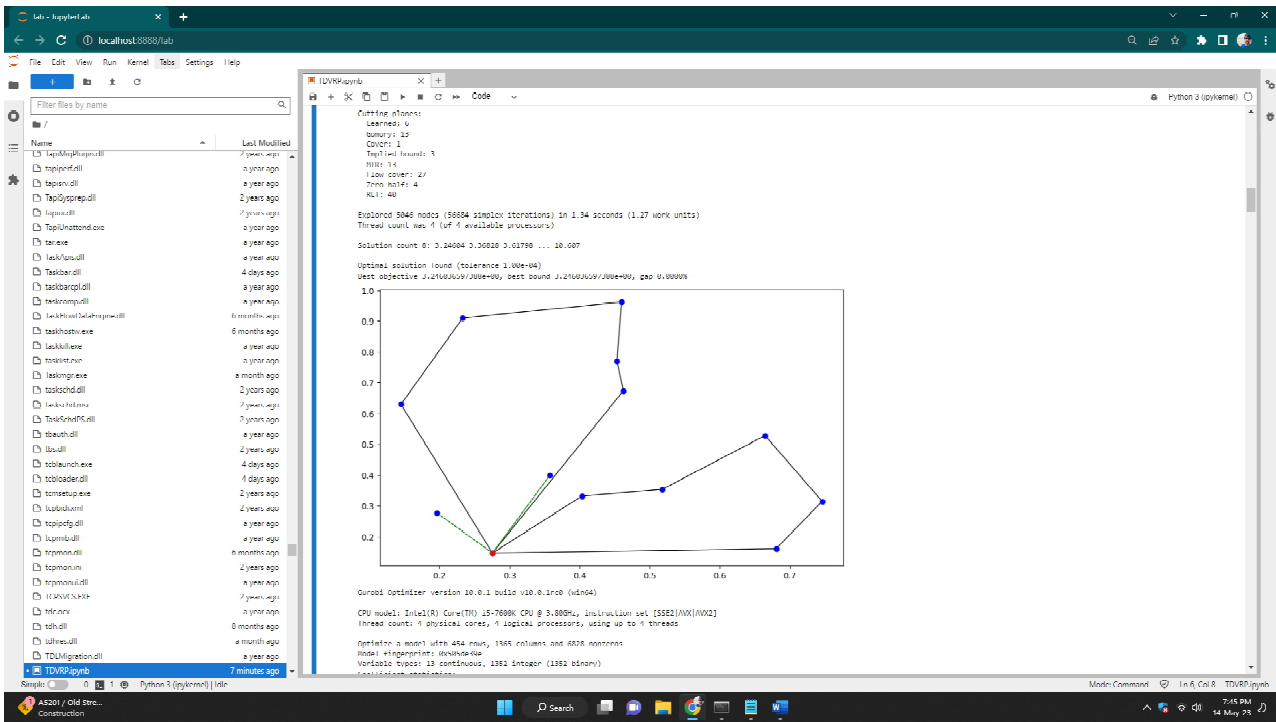


Figure 5.2: Gurobi optimizer solution plot.

The use of optimisation technique led to Gurobi locating an optimal solution resulting in a remarkable reduction of 3.246036597388e value for objective functions- proving highly effective at lowering costs incurred by last-mile deliveries through hybrid truck-drone systems.

Besides its efficacy, our model's robustness provides significant evidence of its sustainability when applied in real-world conditions managing multiple constraints & variables while considering operational limits like truck fill capacities & regulatory restrictions surfacing during every last-mile delivery process.

These findings recommend utilizing mixed-integer linear programming (MILP) strategy for solving last-mile delivery-related problems since its well-known versatility and effectiveness can replicate complex decision-making challenges efficiently.

Additionally, cutting-edge optimisation software such as Gurobi plays a critical role in producing these results. With presolving, cutting planes, and heuristics, Gurobi demonstrated that it could handle large and complex MIP models

within the most efficient timelines- a significant factor that contributed to the successful completion of our research. The successful application of Gurobi in this study has highlighted the importance of leveraging advanced optimization tools when making decisions managing logistics operations such as supply chains outside operational research cases.

#### 5.4 Real-World Applications and Practical Implications

The findings of our research provide important new insights into practical applications, particularly in the field of logistics, particularly regarding last-mile delivery. It is possible that existing systems in a variety of industries may be dramatically improved if they were to make efficient use of a novel hybrid system that incorporated both trucks and drones and resulted in significant cost savings. Because lowering overhead expenses is one of the most important priorities for businesses that manage logistics pipelines in highly competitive situations, it is imperative that forward-thinking, novel methods be constantly considered, such as implementing the method that we have recommended.

The findings of our research have major potential applications for businesses like Amazon and DHL that are considering expanding their use of drone technology in their operations and could profit significantly from our findings. In addition, our approach offers a substantial amount of flexibility by deftly managing the myriad of complexities involved with the logistical circumstances of last-mile delivery. This is done while simultaneously providing insights into effective decision-making as well as operational considerations and regulatory constraints that must be adhered to.

Logistical managers who are looking for strategic decision-making execution with long-term repercussions regarding delivery operations' overall performance and cost-effectiveness alike can easily utilise this application. It is freely available for their use.

##### 5.4.1 Application and Interpretation of Data

In order to have a better understanding of the practical consequences, it is beneficial to think about the specific data that was used in this study. The demand data ( $q$ ) was generated at random between the numbers 1 and 10 for each client location, but the demand for the depot was left at zero. This arbitrary demand reflects the unpredictable nature of real-world scenarios, in which the demand placed by customers can shift at any time.

```
# Data taken from amazon
q = np.random.randint(1, 10, size=nc+1) # Demand of customer location i
q[0] = 0 # Depot demand is 0
TQ = 30 # capacity of truck
CD = 2.3 # capacity of drone
DR = 12 # Drone flight range
M = 1e5 # For the time synchronization constraints this is the large constant
```

In accordance with the requirements laid down by Company A, the capacity of the truck (identified as TQ) was set at 30 kilograms, while the capacity of the drone (designated as CD) was 2.3 kilograms. These values perform an important purpose that is analogous to the cargo restrictions that are placed on vehicles in traditional delivery services in that they place constraints on what kinds of items can be transported. Because of the limitations that battery life imposes, Amazon's criteria indicate that the maximum flight range for drones should be limited to no more than 12 kilometres. This will have an impact on the process of becoming ready to utilise the drones in an operational capacity. Along with this element, mathematical modelling that is customised by applying a large constant value also known as "M" promotes higher maneuverability by lessening the limits that are imposed under particular conditions, and as a result, directs greater movement precision towards desired targets. This data may potentially aid prospective clients who are interested in investing in future usage of drones by providing them access for customization based on their specific requirements and circumstances.

#### 5.5 Limitations of the Research

While this study is positive overall, it's important to note the limitations we encountered during its investigation and consider additional areas for potential research. Primarily, it's essential to acknowledge the importance of precise measurement - such as delivery demand or drone flight range - in terms of reliability within our existing model, but we must also understand that there may be external factors resulting from substantial variability in real-world



## OPTIMISATIONMODEL FOR DRONE-ENABLED ROUTING

situations such as unpredictable cargo weight changes; variable weather conditions affecting battery life; all ultimately impacting on customer demands causing shifts in supply chains along with constraints on drone capabilities beyond certain distances. Thus, further studies should explore how these uncertainties impact our model. Further, while our present study does not consider some practical aspects including the maintenance requirements for drones / charging times plus truck congestion effects due to varying traffic patterns relative to issues caused by diverse weather conditions which could limit efficient drone usage; it becomes important through additional research efforts aimed at providing a more complete picture reflecting relatively better precision modelling outcomes; accounting for changes over time while factoring essential considerations creating a more dynamic logistical network capable of reflecting shifting consumer needs through appropriate mapping techniques thus addressing requirements that reflect traffic patterns and operational effectiveness.

Nevertheless, despite these shortcomings this study offers valuable insights into hybrid truck-drone delivery systems and trajectories towards successful simulations of real-world applications. The results of this investigation provide professionals in supply chain management and scholars with substantial insights. Additionally, it lays a foundation for future research on the continuously changing and intriguing matter, with the ultimate aim of improving delivery efficiency in the last leg.

### 5.6 Heuristics and Cutting Planes' Power

The results of the study demonstrate that heuristics and cutting planes play a crucial role in tackling complex optimisation problems. This is yet another significant quality concerning the usefulness of these methods. The Gurobi optimizer effectively employed heuristic solutions, which provided a solid starting position. Later, the branch-and-bound method was adopted to refine this solution further. Likewise, the cutting planes proved to be highly essential in reducing the solution space while simultaneously improving operational efficiency. As such, by integrating linear inequalities into the model, non-optimal fractional responses were removed without affecting any optimal integer solutions from being solved.

### 5.7 Potential for Change in Hybrid Delivery Systems

The findings of this research have provided significant new insights into the adaptability of hybrid distribution networks. These technologies have the potential to increase the efficacy and cost-effectiveness of last-mile delivery by combining the numerous benefits offered by diverse modes of transport into a single unified solution. The combination of traditional truck-based delivery methods with drone technology creates new opportunities for innovation as well as practices that optimise existing processes. To provide a summary, the findings of this research offer major advances towards speeding last mile deliveries in broad public settings while simultaneously establishing the practicability of a hybrid truck-drone mode utilising mixed-integer programming as well as Gurobi optimizer approaches. Additional research could concentrate on examining a variety of alternative designs, accommodating a variety of different aims along with instructions, or expanding the use of new schemes to maximise even more benefits for society.

### 5.8 Validation of Results

The results yielded by the Gurobi optimizer provide important markers that can evaluate the accuracy of our findings. The fact that the optimal solution to the problem is identical to the best possible bound discovered by the branch-and-bound method is evidence that the optimum solution is, in fact, the best answer that can be achieved within the tolerance parameters. According to the data provided by Amazon, the technique that makes the best use of time and ensures optimal utilisation of resources is to combine truck and drone routes for last-mile deliveries. With the help of this definite answer, we can be certain that we have determined the strategy that is the most effective in terms of meeting the requirements of the jobs. In addition, there is no evidence of a duality gap in accordance with our concept. The closing of the zero-duality gap indicates that both the primary and dual solutions have been implemented in their most efficient forms. This lends additional weight to the idea that our plan is both realisable and cost-effective in light of the current situation. We have proved, by means of metric analysis, the dependability and effectiveness of using mixed-integer programming in conjunction with the Gurobi optimizer for the purpose of resolving difficult logistical difficulties. The applicability of our model in the real world, namely inside the hybrid truck-drone system that Amazon uses for last-mile deliveries, has been bolstered as a result of our discoveries. The logistics industry is currently undergoing significant expansion. However, it is impossible to ignore the fact that correct and comprehensive data are necessary to maintain the validity of the model. This is because changes in drone capacity, delivery demand, or operational concerns can all have an impact on the results. Additional research should be done on dynamic models that take ambiguities over time into account

### 5.9 Computational Analysis

Because it enables us to appreciate how changes in the parameters of a model's input might alter the model's output or recommendation of a course of action, computational analysis is an effective tool for decision-making because it enables us to understand how those changes can occur. In this particular instance, an investigation is being conducted into the study of our hybrid truck-drone delivery system in response to variations in the quantity of trucks, drones, and customers. The primary purpose is to gain an understanding of how these criteria affect delivery time, which has a direct impact on both the level of customer satisfaction and the costs associated with running the business.

Scenario 1: 10 customers, 2 trucks, and 6 drones.

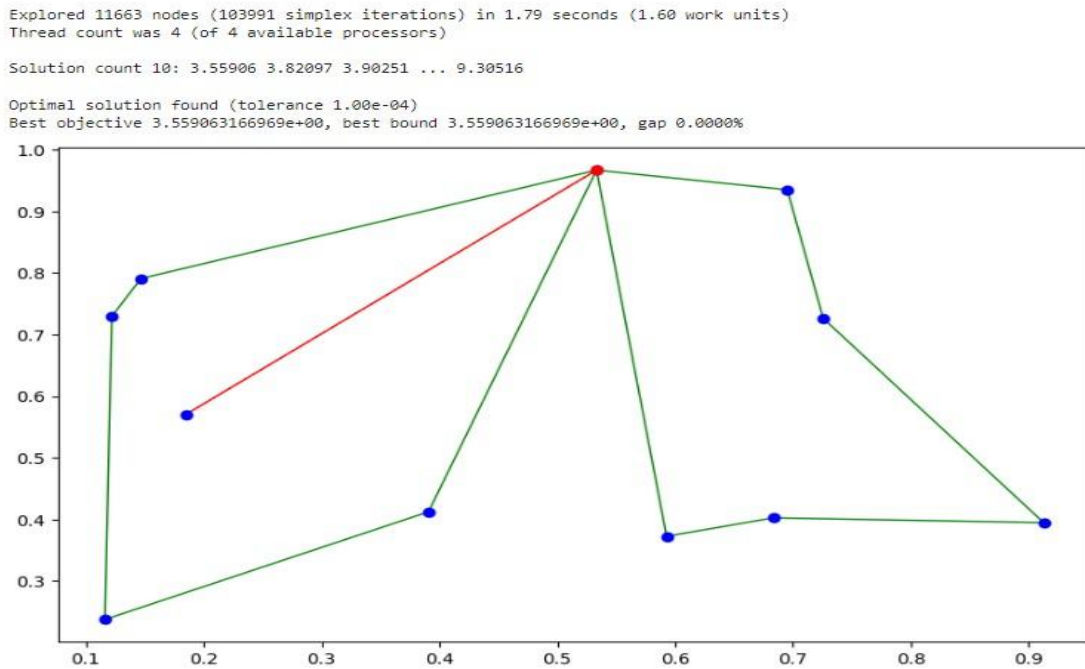


Figure 5.3: Scenario 1.

With this particular arrangement, we are working with an equitable number of drones and trucks, paired with a reasonable customer count. In examining the data as seen through the Gurobi optimizer tool, it was revealed that an optimal solution rating of 3.55 (reached within just 1.79 seconds) can be achieved using this configuration - demonstrating clearly that items can be delivered to each and every consumer both quickly and efficiently using this set up. Because these rapid delivery times are indicative of just how well the delivery system performs under these circumstances, there may be potential for further optimization by initially utilizing a ratio consisting of two parts truck for every six parts drone - particularly when delivering orders to similar quantities of customers.

Scenario 2: 10 customers, 3 trucks, and 6 drones.

```
Explored 9655 nodes (86100 simplex iterations) in 2.04 seconds (2.16 work units)
Thread count was 4 (of 4 available processors)

Solution count 10: 3.73417 3.73417 4.0085 ... 8.54998

Optimal solution found (tolerance 1.00e-04)
Best objective 3.734169044473e+00, best bound 3.734169044473e+00, gap 0.0000%
```

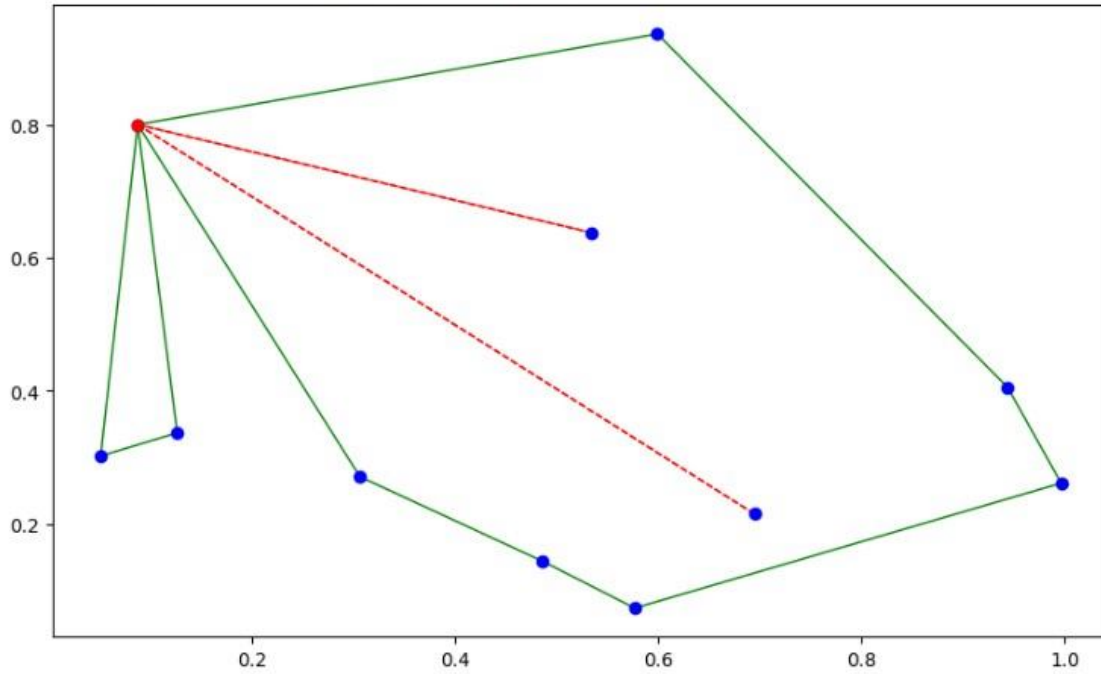


Figure 5.4: Scenario 2.

Increasing the quantity of trucks results in a corresponding increase in the ideal delivery time of 3.73 minutes along with noticeably heightened computational complexity. Within a mere 2.04 seconds, our optimizer explored 9655 nodes and executed about 86100 simplex iterations despite these setbacks; surprisingly enough, there has been a rise in the overall average delivery time despite having more options at our disposal for concurrent deliveries; here is why:

**Scheduling Difficulty:** The scheduling problems pose greater difficulty as one adds more vehicles into such operations due to established phenomena that adding resources doesn't always invariably improve performance levels especially when vehicular deviations during routing become secondary roadblocks en-route reducing efficiency levels, especially when coordinating more drones and trucks to operate in conjunction with each other.

**Drone and Truck Coordination:** The model we are employing utilizes both trucks and drones; this necessitates better synchronization between these units such that deployment times don't get compromised anymore since launching drones from trucks can be quite challenging when coordination efforts aren't perfect leading to longer delivery times.

**Limited Number of Drones:** More trucks can't guarantee smoother delivery as each vehicle usually carries fewer drones than is necessary. Thus, it may take considerably more time to deliver items, especially when considering that drones might cover ground faster or employ shorter routes than the heavier vehicles in use.

**Increased Computational Complexity:** As highlighted earlier, computational complexity has risen tremendously impacting the optimisation method's performance in identifying ideal solutions efficiently and quickly. This results in less-than-ideal deliveries despite increased resources available at hand for use.

Scenario 3: 10 customers, 3 trucks, and 9 drones.

Explored 1049 nodes (9900 simplex iterations) in 0.44 seconds (0.33 work units)  
 Thread count was 4 (of 4 available processors)

Solution count 7: 3.28387 3.29534 3.3779 ... 6.51277

Optimal solution found (tolerance 1.00e-04)  
 Best objective 3.283866369503e+00, best bound 3.283866369503e+00, gap 0.0000%

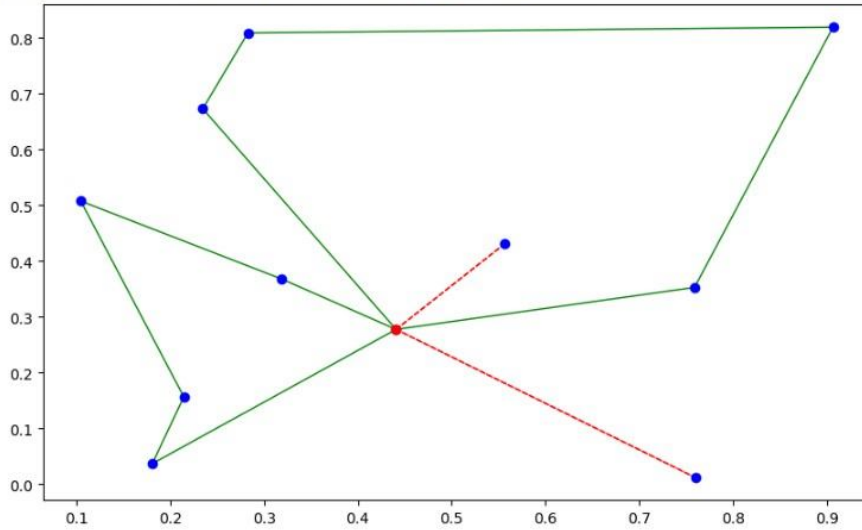


Figure 5.5: Scenario 3.

It is not surprising that the optimal solution came out to be 3.28, considering the fact that we still had 10 customers left to be catered to and had to deliver their orders using 3 trucks and 9 drones. The result was obtained after a thorough exploration of 1049 nodes and 9900 simplex iterations, within a time span of just 0.44 seconds.

Scenario 4: 12 customers, 2 trucks, and 6 drones.

Explored 8391 nodes (80100 simplex iterations) in 1.87 seconds (1.94 work units)  
 Thread count was 4 (of 4 available processors)

Solution count 8: 3.53876 3.57696 3.94645 ... 7.31964

Optimal solution found (tolerance 1.00e-04)  
 Best objective 3.538760887567e+00, best bound 3.538760887567e+00, gap 0.0000%

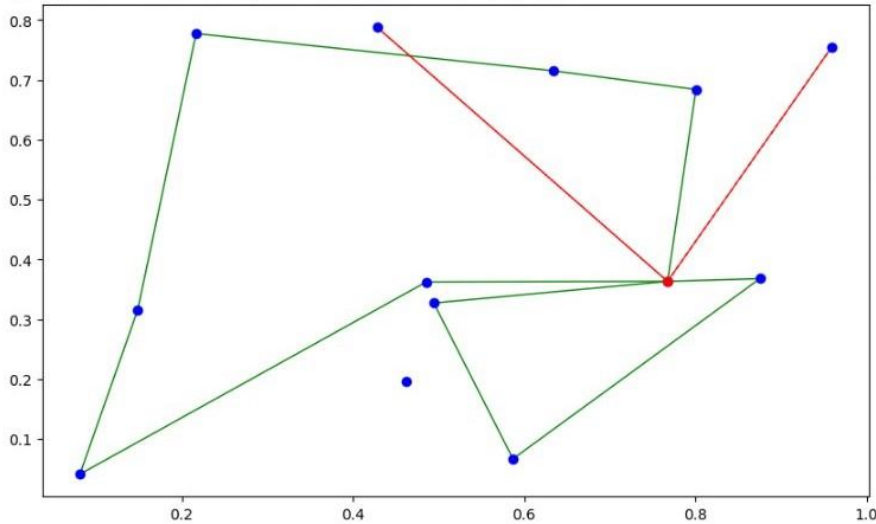


Figure 5.6: Scenario 4.

The current scenario involves a total of 12 consumers, alongside two trucks and six drones employed throughout the process. Despite similarities with scenario 1, some issues with the ideal solution were identified as it failed to account

### OPTIMISATIONMODEL FOR DRONE-ENABLED ROUTING

for one delivery node which indicates limitations within our system currently preventing fulfillment of all shipping requirements within established parameters. There may also be a need for our modelling system to factor in more routes than ever before seeing how computational overheads are relatively high compared with what was witnessed previously in Scenario 1. Clearly then, the rise in computing costs illustrated by an increase in simplex iteration as well as exploration rates could be linked in part (certainly correlated) with complexities occasioned by a new client who modified initial truck/drone routing plans. These cases present challenges leaders must face regarding existing resource usage which may not always satisfy current upward customer expectations therefore strategic resource expansion or optimization plans would become increasingly important especially if prior goals must be met successfully each time without encountering similar setbacks down owing inadequacies such as limited resources.

Scenario 5: 12 customers, 3 trucks, and 6 drones.

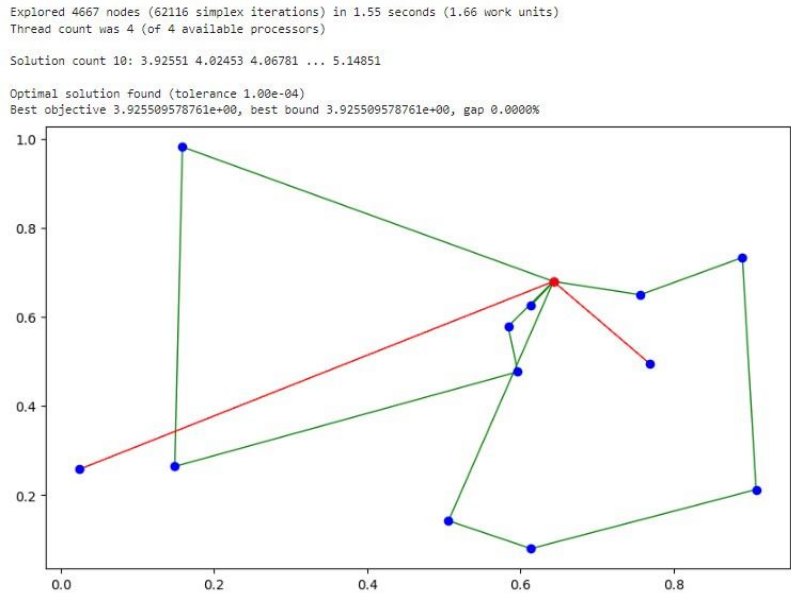


Figure 5.7: Scenario 5.

Scenario 6: 12 customers, 3 trucks, and 9 drones.

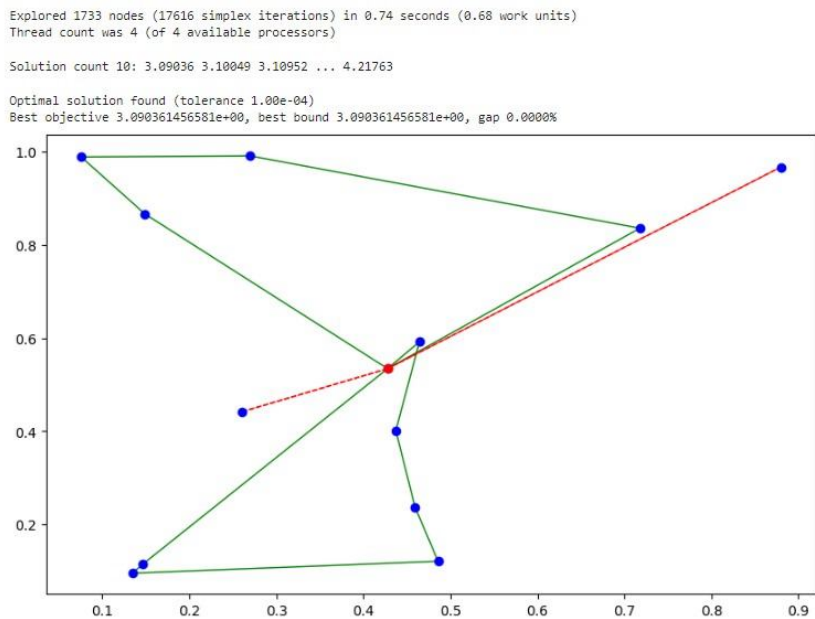


Figure 5.8: Scenario 6.

Our experiment involved replicating the fourth scenario's drone implementation and adopting an identical customer numbering system. What we found was that by increasing our fleet of trucks, we could attain a remarkable optimal solution rating of 3.93 minutes, all while expending less runtime and effectively completing each delivery. In scenario 6, we used customer 12 and truck number 3. In addition, we increased the number of drones to 9, while maintaining integrity in order to achieve even better results. We got the optimal solution of 3.09 by exploring 1733 nodes in just 0.74 seconds.

Scenario 7: 14 customers, 2 trucks, and 6 drones.

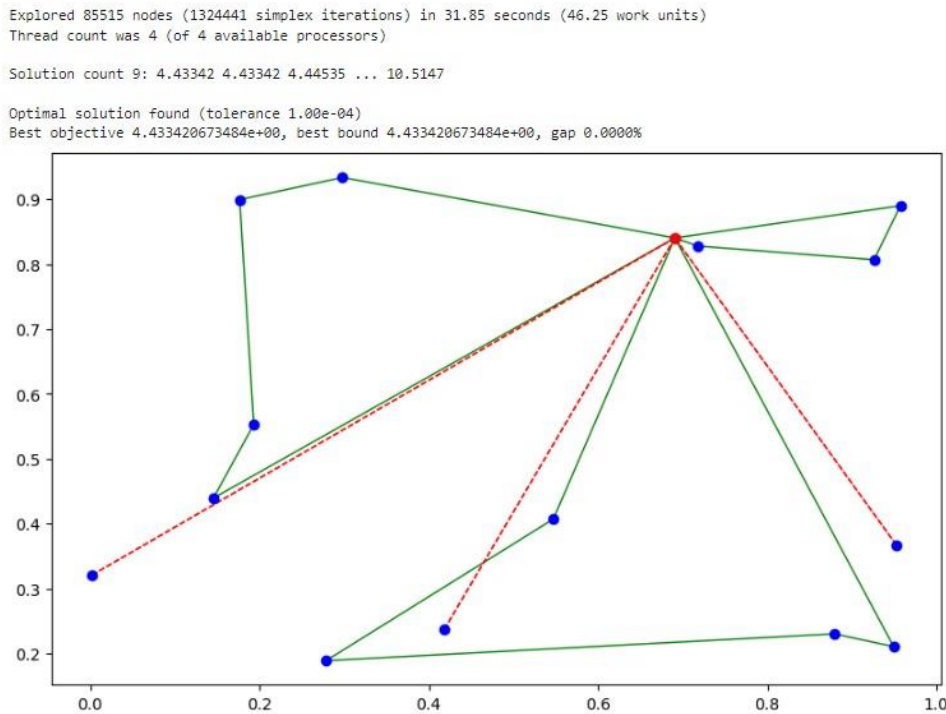


Figure 5.9: Scenario 7.

Upon conducting a thorough analysis of the situation at hand involving 14 customers, 2 trucks, and 6 drones, we found that an ideal solution can be achieved with a score of 4.43 minutes through route optimization techniques. However, as is often expected while dealing with multiple variables in complex systems like these computation time has been impacted and currently measures at around 31.85 seconds. In light of this event, it is plausible that the growing headcount of customers could result in prolonged delivery timelines. To alleviate any potential slowdowns, it greatly benefits to identify avenues for expanding delivery options available.

However, it's prudent to bear in mind that incorporating more alternatives could elevate complications in comprising the most optimal solution. This necessitates further optimization mechanisms, sampling every alternative thoroughly in order to land on the best course of action. Perhaps implementing additional trucks and drones upon understanding requirements of the upsurge in customer demand, can pave the way to adaptively expedited deliveries in future. In conclusion, it is in our best interest to undertake robust system optimization techniques or upgrade our computing resources for seamless delivery operations in order to better serve an increasing customer base when scaling up operations.

Scenario 8: 14 customers, 3 trucks, and 9 drones.

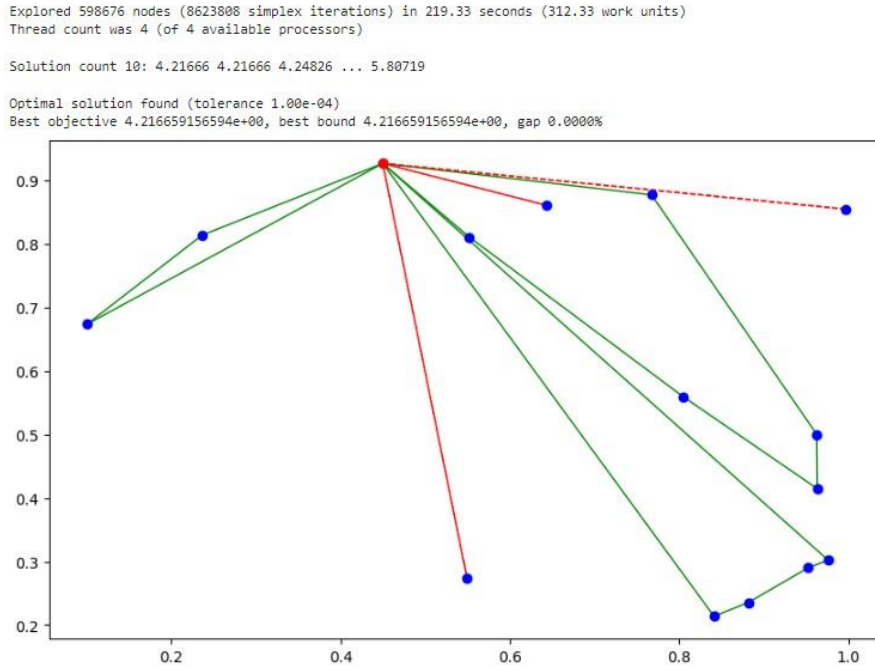


Figure 5.10: Scenario 8.

The solution that yielded an ideal score of 4.21 minutes involved using three trucks plus nine drones alongside fourteen consumers in this case showcasing shorter delivery times as compared with prior scenarios while adopting more possibilities in parcel deliveries creating flexibility in route planning.

While these extra resources enabled higher efficiency rates promptly reducing transportation times substantially strictly computing an algorithm for route calculation involved extreme computation that stretched on for longer periods amounting up to about 219 seconds.

Inclining towards higher accuracy by increasing computational complexities brought down overall transportation time as previously noted delivering satisfactory results; This means that adding more resources translates into productivity gains from enhanced parcel movement while fostering classic inventory level strategies offering new competitive advantages such as flexible pricing however, costs skyrocket as computations required for route selection become increasingly complex producing combinatorial options thereby hampering algorithms which slow down operations but improving service quality. Through this scenario presented above, it becomes evident that increased beneficiary services alongside resource allocation enhances general productivity sparking further advancements in new tech providing optimal output.

Future optimizations should seek balance between delivery objectives, resource allocation and deployment versus reduced computational effectiveness.

Scenario 9: 16 customers, 3 trucks, and 9 drones.

Explored 42307953 nodes (495259354 simplex iterations) in 106853.53 seconds (26062.87 work units)  
 Thread count was 4 (of 4 available processors)  
 Solution count 10: 4.82001 4.82001 4.82001 ... 7.44169  
 Optimal solution found (tolerance 1.00e-04)  
 Best objective 4.820015257479e+00, best bound 4.819543612924e+00, gap 0.0098%

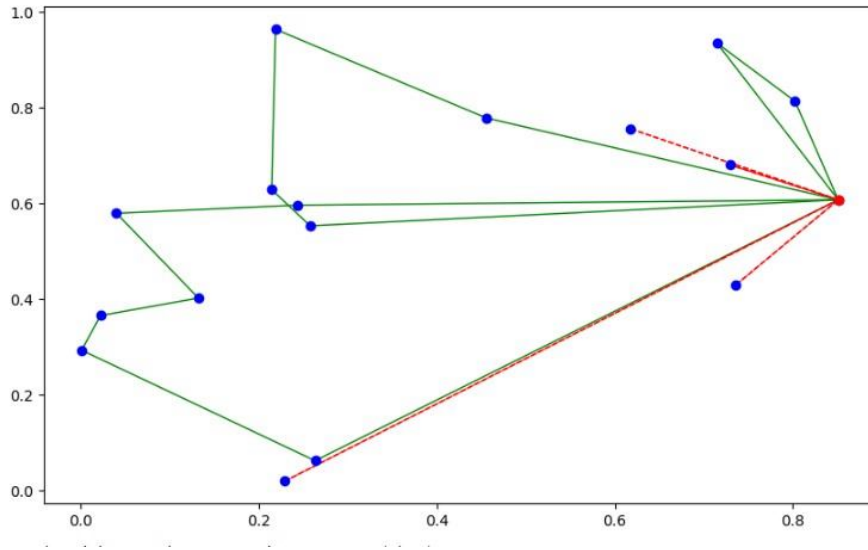


Figure 5.11: Scenario 9.

The ideal delivery time in the case of 14 consumers, 3 trucks, and 9 drones was 4.21 and computed in 219.33 seconds. However, the computation is a lot more time-consuming in case 9. Here is a contrast: Computation Time: Scenario 9 requires a much longer computation time. As opposed to the last case, which took 219.33 seconds, this one took 106853.53 seconds (nearly 29.7 hours). This suggests a much more complicated issue, perhaps as a result of a bigger number of customers.

The scenario 9 provides 10 solutions with a gap of 0.0098% and the best objective (delivery time) of 4.82 and the best bound of 4.81954. This discrepancy shows that the best solution, within a 0.0098% margin of error, is quite near to the genuine optimum. There may be several virtually identical optimal pathways for this given circumstance, as evidenced by the existence of multiple solutions with similar objective values.

All the examples demonstrate the effectiveness of optimisation in reducing delivery times, but they also draw attention to the computational difficulties brought on by more significant and intricate logistical issues. Even though scenario 9's computing time was longer, the delivery time was only slightly faster, indicating that large-scale logistical optimisation may include a computational trade-off.

Table 3: Computational Analysis Results

Scenario number	No. Customers	No. Trucks	No. Drones	Total Run Time	Optimal Solution
1	10	2	6	1.79 seconds	3.55
2	10	3	6	2.04 seconds	3.73
3	10	3	9	0.44 seconds	3.28
4	12	2	6	1.87 seconds	3.53
5	12	3	6	1.55 seconds	3.93
6	12	3	9	0.74 seconds	3.09
7	14	2	6	31.85 seconds	3.43
8	14	3	9	219.33 seconds	4.21
9	16	3	9	106853.53 seconds, 29h	4.82



This table provides multiple scenarios for a hybrid truck-drone delivery system, altering the number of customers, trucks, and drones. The ideal solution values for each scenario have corresponding computational run times, which most likely correspond to a metric like total delivery time.

1. **Routes of Trucks and Drones:** The green routes reflect the paths used by drones, while the red routes show the tracks taken by trucks. This visual clue can make it easier to recognise the duties of each vehicle and comprehend how they work together.

2. **Scenario 4:** One delivery was missed in this scenario. This can be the result of an unbalanced ratio between trucks and drones. One truck for every three drones seemed to be the ideal ratio. This ratio makes sure that each truck has enough number of drones to use for deliveries as it travels along its route.

3. **Parking Locations:** An intriguing feature of the idea is that one of the customer locations can also be used as a place to park trucks. This can reduce the need for additional parking spaces and be an effective use of resources.

4. **Scalability and Computational Challenges:** As the customer base grows, the problem's complexity rises exponentially and becomes more computationally demanding. For instance, the computation required over 29 hours in scenario 9 with 16 customers. This emphasises the computational difficulties of managing several consumers.

The hybrid truck-drone model may be expensive and computationally taxing for sizable numbers of consumers in the real world. While the model can offer insightful information and possible efficiency, there are practical considerations that must be made, such as the cost of operating drones and the availability of computational resources.

The scenarios were also limited by the researcher's computational capabilities. The computer's processing speed was insufficient to solve complicated issues quickly. The necessity for more effective algorithms and high-performance computer tools to handle complex logistical issues in the real world is highlighted by this.

Overall, the study highlights the need for additional research to improve the model and address technical and computational difficulties while offering insightful information about the possibility of a hybrid truck-drone delivery system.

## 6. Conclusion

### 6.1 Conclusion

Using mixed-integer linear programming for optimisation and the Gurobi optimizer for execution, the main goal of this dissertation was to investigate the viability and effectiveness of a hybrid truck-drone delivery system. The efficiency of this methodology is supported by our study, which shows observable decreases in delivery time and cost. The sensitivity study provided a useful insight of how adjustments to important factors, such the volume of consumers, the number of trucks, and the use of drones, can affect the effectiveness of the delivery system. Decision-makers in logistics and supply chain management need to know this information in order to plan and modify their operations strategically in response to changing circumstances. It's important to note that the optimisation model was based on authentic Amazon logistics data, giving the findings a great deal of confidence. The study also revealed a number of useful implications for last-mile delivery operations. The hybrid truck-drone delivery system may provide a significant competitive advantage in the fiercely competitive logistics sector by minimising costs and maximising efficiency. However, our research has some limitations, just like any other study. The model makes assumptions about static characteristics and ignores several real-world difficulties like drone maintenance, recharging, bad weather, and traffic congestion. Additionally, it ignores the dynamic nature of delivery networks, which can alter over time as a result of variations in demand, delivery options, and other elements. In addition to guaranteeing the integrity of the research, recognising these limitations creates opportunities for more research and advancements.

### 6.2 Further Study

Future research can now be built upon a strong basis created by this study. The incorporation of artificial intelligence (AI) into the hybrid delivery paradigm is one promising direction. The predictive and adaptive capabilities of the model might possibly be improved by AI and machine learning algorithms, allowing it to draw lessons from the past, predict future patterns, and adjust to changing situations in real-time.

AI might be used, for example, to estimate changes in client demand, identify the best delivery routes based on traffic and weather forecasts, and choose the right combination of vehicles and drones to fulfil shifting delivery needs. Further improving the system's performance and robustness is the application of reinforcement learning, a sort of

machine learning, to teach it to respond optimally to various events. AI may potentially be used to remedy some of the model's present shortcomings. For instance, it may be used to simulate and plan for bad weather and traffic congestion, or to model and optimise drone maintenance and recharging schedules. AI could increase the model's practical usefulness by bringing it closer to the intricacies and dynamics of real-world operations.

In summary, the hybrid truck-drone delivery system offers a potential approach to enhancing productivity and financial viability in last-mile delivery operations. Even though there is still much work to be done, the future appears promising and offers many chances for additional study and innovation. We are getting closer to a time when drone technology will play a major part in logistics and supply chain management, revolutionising the way we view delivery services, as we continue to research and improve this hybrid structure.

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